GENERAL PHYSICS I
(MECHANICS)

by

Bassam SAQQA & Nasser FARAHAT

The Islamic University of Gaza

THIRD EDITION
(2006)

بسم الله الرحمن الرحيم

تقديـــــــــــــم

درجت كثير من الجامعات في العالم العربي على تدريس مساق الفيزياء العامة من الكتب و المراجع الأجنبية. وقد وجد كثير من الدارسين والأكاديميين أن ذلك يقلل من مدى تحصيل الطالب و استيعابه، نظرا لهدر جل وقته في ترجمة وفهم التعبيرات الإنجليزية المستخدمة. لذلك فقد ظهر فريق من الأساتذة و الأكاديميين يدعون إلى تعريب التعليم الجامعي، وبدأوا في ترجمة أو تأليف الكتب والمراجع الجامعية باللغة العربية. ولا شك أن استيعاب الطالب بلغته الأم أكثر من استيعابه باللغة الإنجليزية. وقد انقسم الأكاديميون إلى فريقين، فريق يدعوا إلى التعريب وفريق يدعوا إلى بقاء التعليم في الكليات العملية والتطبيقية باللغة الإنجليزية، ولكل فريق حجته ووجهة نظره.

فريق الأكاديميين الذين يدعون إلى تعريب التعليم فحجتهم في ذلك أن العربية من اللغات الحية و التي يمكن أن تكون لغة علم وتستطيع استيعاب كل المسميات والمصطلحات العلمية. كذلك فان مدى استيعاب الطالب للمادة العلمية يكون أكثر وأعمق، وهم مصيبون في ذلك.

وفريق الأكاديميين الذين يعارضون التعريب حجتهم أنه في التعريب سينشغل الأكاديميون في ترجمة المراجع وينصرفون عن الإبداع و الاكتشافات، إضافة إلى أن الدراسة باللغة العربية تعمل على خلق فجوة علمية عند مواصلة الطالب دراسته في الدول الأجنبية، فيبقى الطالب شاعرا بالنقص، ولا يستطيع مجاراة زملائه في الدراسات العليا، وهم مصيبون في ذلك أيضا.

ونحن نرى أنه عندما يصبح لدينا الإنتاج العلمي والتقدم التكنولوجي المضاهي والمنافس لتكنولوجيا الغرب، عند ذلك يجب أن يكون كل إنتاجنا العلمي باللغة العربية الأصيلة مما يجعل العالم يترجم عن العربية هذا الإنتاج العلمي، وهذا ما حصل مع بداية النهضة الصناعية الأوروبية في نهاية القرون الوسطى، وبلوغ الحضارة العربية الإسلامية ذروتها.

أما في عصر التخلف العلمي والتقني الذي نعيش، فإننا نقول وبكل أسف أننا يجب أن نبقى نعتمد اللغة الإنجليزية في الدراسة والتدريس. وحتى نتغلب على الصعوبات التي يواجهها طلابنا من استخدامهم المراجع الأجنبية والمؤلفة خصيصاً للطلاب الناطقين بالإنجليزية أصلاً، كان هذا الكتاب.

كتب هذا الكتاب لمساق الفيزياء العامة (أ) بلغة إنجليزية بسيطة، وبتعبيرات سهلة ميسرة، تتناول المفاهيم الأساسية بعيدا عن التطويل والتكرار، تتناسب مع الطالب الذي أخذ قسطا من الإنجليزية في دراسته الإعدادية والثانوية.

يهتم مساق الفيزياء العامة (أ) بعلم الحركة أو "الميكانيكا" و هو من العلوم التي تقوم عليها مجريات الحياة اليومية التي نعيش. وقد رأينا أن نبدأ بدراسة موضوع الكميات المتجهة و الكميات القياسية في الباب الأول لإيماننا بأهمية هذا الموضوع في فهم باقي المواضيع في الأبواب اللاحقة. أما الباب الثاني فخصص للحركة في بعد واحد ثم في بعدين في الباب الثالث.

الباب الرابع والخامس خصصا لقوانين نيوتن المشهورة للحركة وتطبيقاتها و الباب السادس والسابع لنظرية الطاقة والشغل وحفظ الطاقة. وفي الباب الثامن تم دراسة كمية التحرك والتصادم، وفي الباب التاسع الحركة الدورانية، والباب العاشر علم السكون والاتزان، وفي الباب الحادي عشر الحركة التذبذبية، وختم بالباب الثاني عشر في ميكانيكا الموائع.

ووضع بعد شرح القوانين الرياضية ومعانيها الفيزيائية عدة أمثلة توضيحية تشرح للطالب في المحاضرة، وبعد كل باب وضعت مجموعة مناسبة من المسائل والتمارين، حتى يقوم الطالب بحلها بنفسه. ووضعت أجوبة كل المسائل في نهاية الكتاب.

في نهاية الكتاب وضعت عدة ملاحق يحتاجها الطالب في دراسته. وختاما نسأل الله العلي القدير أن نكون قد وفقنا في عملنا هذا، ونحن مع كل نقد وتصويب من زملائنا المدرسين، وكذلك الطلبة، حتى نعمل على إنتاج الأحسن بإذن الله، والله من وراء القصد.

المؤلفان

غزة في أغسطس ١٩٩٨

PREFACE TO THE FIRST EDITION

From our experience in teaching general physics for the past few years at the Islamic University of Gaza we have faced with a major handicap: the text book. Although many excellent books are available, non of them, in our opinion, is appropriate for our students who came from secondary schools with limited background in English. Since most of the available books are written for English-speaker students, our students find difficulty in reading and understanding the physical concepts from these books. With these remarks in mind, we encouraged by our colleagues to write a text book in a simple language such that our students can easily handle. To achieve this goal we tried to minimize the long-term explanations as possible as we can and summarize the concepts and the ideas of the subject in an obvious form such that the student can find the important ideas without going in the unnecessary details and explanations.

This book is intended for a one-semester course in general physics (part A) for students majoring in science and engineering in their first year in university. The material of our book is written in a manner to be appropriate for students knowing some calculus enough to differentiate and integrate simple calculus. Concurrent study of introductory course in calculus would be highly recommended.

The first chapter is an introductory chapter dealing with unit systems and vectors. Vectors play an important role throughout mechanics. Consequently, all the necessary concepts concerning vectors , including the scalar product and the vector product are put in this chapter.

The motion of a particle in one dimension is studied in chapter 2, while chapter 3 is devoted to the motion of a particle in two dimensions including a section in relative motion and another in circular motion.

Newton’s laws of motion are treated in chapter 4, followed by a more detailed discussions of circular motion in chapter 5 which also included gravitation.

Chapter 6 is concerned with the concepts of work and energy, and chapter 7 with the concept of conservation of energy. Conservation of linear momentum and collision is introduced in chapter 8.

Rotational motion is developed in chapter 9 with the concepts of torque and conservation of angular momentum. Chapter 10 deals with static equilibrium of a rigid body and elasticity.
The oscillatory motion is introduced in chapter 11, while the final chapter is devoted to fluids mechanics.

There is absolutely no substitute for attacking and solving problems to test your understanding of the principle of physics. Because of that a number of solved examples are given in each chapter to be solved by the teacher in the class. These examples are chosed to serve as models for the problems at the ends of the chapters. About twenty problems are given at the end of each chapter. We tried to write these problems in a simple language such that our students can easily understand. Simple diagrams are drawn for most of the problems. Answers of all the problems are given at the end of the book.

The authors wish to express their thanks to all the members of physics department who encourage us to write this text book.

B. Saqaa
N. Farahat
Gaza,
August 1998

PREFACE TO THE THIRD EDITION

Since the editing of the first edition in 1998 we have received several comments and remarks from those who used this book as a text book and from the students along many years. Most of these remarks are positive when speaking about the idea of writing a text book for Non-English-speaker students. Many suggestions are also made from professionals regarding some chapters and some sections.

In view of such suggestion from the users of the book and the positive criticism from the reviewers of the manuscript many changes are made in this third edition. Those changes are:

1- Many new problems are added to each chapter.

2- In some chapters we added new examples to attack more ideas such that the students can understand the concepts of the chapter in a proper way.

3- The concept of relative motion is addressed in a new way and is discussed in both chapter 2 and chapter 3. In chapter 2 we discussed the relative motion in one-dimension, while in chapter we considered the relative motion in two-dimensions.

4- The concept of work discussed in chapter 6 is displayed in a new manner to be more clear and understandable for the students.

5- Some changes are made in writing chapter 7 where we discussed the two types of potential energy before going to the concept of conservation of energy.

6- Chapter 8 is completely changed where we divide the topics of the chapter in two chapters: chapter 8 is devoted to the concept of Systems of Particles and in chapter 9 we discussed the concept of collision.

7- A new section is added to chapter 10 with the name Work and Rotational Motion.

We wish to thank all the readers of the book and the reviewers of the manuscript for their positive criticism and helpful suggestions. Special thanks go to our collogues in the department of physics at the Islamic university of Gaza and the teaching assistants in the departments who make most of the misprinting errors.

B. Saqaa
N. Farahat
Gaza,
August 2006

CONTENTS

CHAPTER 1

MEASUREMENTS AND VECTORS

1.1 UNITS AND STANDARDS

Any physical quantity must have, besides its numerical value, a standard unit. It will be meaningless to say that the distance between Gaza and Jerusalem is 80 because 80 kilometers is different from 80 meters or 80 miles, where kilometer, meter, and mile are standards for length known all over the world. Several systems of units are used in physics: The most common system among them is the **Systém International** (French for International System) abbreviated **SI**. The SI units for the seven-fundamental physical quantities are ***kilogram*** for mass, ***meter*** for length, ***second*** for time, ***kelvin*** for temperature, ***ampere*** for electric current, ***candela*** for luminous intensity, and ***mole*** for the amount of substance. All other physical quantities are derived from these basic quantities. In mechanics all quantities are derived from the three-fundamental quantities: **mass**, **length**, and **time**.

One meter *is defined as the distance traveled by light in vacuum during a time of 1/299,792,458 second.*

One kilogram *is defined as the mass of a specific platinum-iridium alloy cylinder kept at the International Bureau of Weights and Measures in France.*

One second *is defined as the time required for a cesium-133 atom to undergo 9,192,631,770 vibrations.*

The other two systems are the **Gaussian system** and the **British engineering system**. The three systems and their standard units for mass, length, and time are listed in **Table 1.1.** Because of their units the first two systems are sometimes called the ***mks*** system (the first letters of meter, kilogram, and second) and the ***cgs*** system (the first letters of centimeter, gram, and second) respectively.

If different unit systems are used in a physical equation one system should be chosen and the quantities with the other unit systems must be converted to the chosen system according to the conversion factors of **Table. 1.2.**

Table 1.1 Units of Length, Mass, and Time in Different systems.

Systems	Length	Mass	Time
SI	meter (m)	kilogram (kg)	second (s)
cgs	Centimeter (cm)	gram (g)	second (s)
British	foot (ft)	slug	second (s)

Table 1.2 Conversion Factors.

Length	Mass	Time
1 m=10^2 cm=3.28 ft	1 kg =10^3 g	1 year=3.16x10^7 s
1 mi=5280 ft=1.61 km	1 slug=14.6 kg	1 day=8.64x10^4 s
1 yd=3 ft=36 in	1 u=1.66x10^{-27} kg	

1.2 DIMENSIONAL ANALYSIS

Dimension in physics gives the physical nature of the quantity, whether it is a length (L), mass (M), or time (T). All other quantities in mechanics can be expressed in terms of these fundamental quantities. For example, the dimension of velocity is length divided by time and denoted by $[v]=\frac{L}{T}$. The dimensions of some physical quantities are listed in **Table 1.3.**

Table 1.3 Dimensions and units of some physical quantities

Quantity	Dimension	Unit (SI, cgs, British)
Area	L^2	m^2, cm^2,ft^2
Volume	L^3	m^3, cm^3, ft^3
Velocity	L/T	m/s, cm/s, ft/s
Acceleration	L/T^2	m/s^2, cm/s^2, ft/s^2
Force	ML/T^2	Newton (N), dyne, pound
Energy	ML^2/T^2	Joul (J), erg, ft.lb
Power	ML^2/T^3	Watt (W), erg/s,

Example 1.1 Show that the equation $x = v_o t + \frac{1}{2}at^2$ is dimensionally correct.

Solution Since $[x]= L$, $[v_0]= \frac{L}{T}$, and $[a]= \frac{L}{T^2}$,

Then

$$L = \frac{L}{T}T + \frac{L}{T^2}T^2 = L$$

1.3 VECTORS AND SCALARS

A **scalar** is the physical quantity that has magnitude only, for example, time, volume, mass, density, energy, distance, temperature. On the other hand a **vector** is the physical quantity that has both magnitude and direction, for example, displacement, velocity, acceleration, force, area.

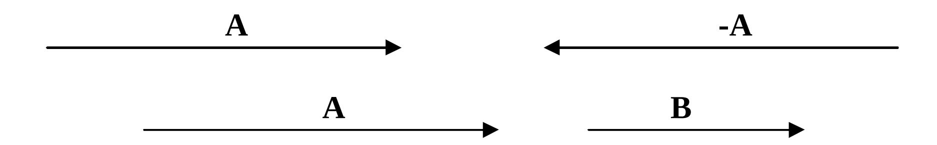


Figure 1.1 Equality of vectors and the negative of a vector. Note that $A>B$.

The vector quantity will be distinguished from the scalar quantity by typing it in boldface, like **A**. In write handing the vector quantity is written with an arrow over the symbol, such as, $\vec{A}$. The magnitude of the vector **A** will be denoted by $|\mathbf{A}|$, or simply the

italic type *A*. Geometrically, The vector quantity is represented by a straight line and an arrow at one end of the line. The end at which the arrow is attached is called the **head of the vector**, while the other end is called the **tail of the vector**. The length of the line is proportional to the magnitude of the vector and the arrow points toward its direction (see **Figure 1.1)**.

Equality of Two Vectors Any two vectors are said to be equal if they have the same magnitude and point in the same direction, regardless of their location.

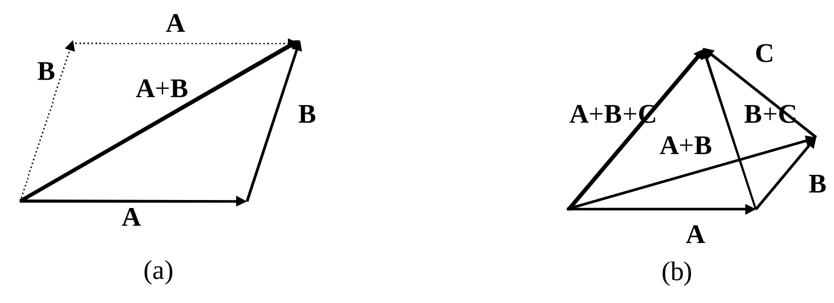


Figure 1.2 (a) The addition of vectors graphically showing the commutation law. (b) The distribution law of addition.

Addition (graphical method). To add vector **A** to another vector **B**, (1) in a graph paper, draw vector **A** with its magnitude represented by a proper scale. (2) Draw vector **B** according to the same scale such that its tail starts at the head of vector **A**. (3) Draw the resultant vector **R**=(**A**+**B**) from the tail of vector **A** to the head of vector **B,** (see **Figure 1.2**). This method is known as the triangle method.

Another method is the parallelogram method. In this method the tail of the two vectors **A** and **B** are coincident and the resultant vector is the diagonal of the parallelogram formed by the two vectors **A** and **B**. As it is clear from **Figure 1.2**, the addition of vectors obey the commutation and association laws, i.e., respectively

$$\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A} \tag{1.1}$$

and

$$\mathbf{A}+(\mathbf{B}+\mathbf{C})=(\mathbf{A}+\mathbf{B})+\mathbf{C} \tag{1.2}$$

Negative of a Vector The vector **-A** is a vector with the same magnitude as the vector **A** but points in opposite direction.

Subtraction Subtracting vector **B** from vector **A** is the same as adding **-B** to **A**, i.e.,

$$\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B}) \tag{1.3}$$

The arithmetic processes of vectors using the graphical method described above are not easy. In the following sections we describe easier techniques that involve algebra.

1.4 UNIT VECTOR

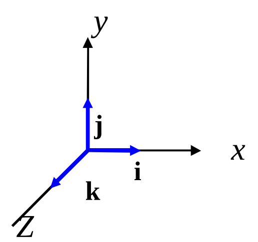


Figure 1.3 The unit vectors **i**, **j**, and **k** are pointed toward the positive *x*, *y*, and *z* axes, respectively.

From its name, a unit vector is a vector, with specific direction, having a magnitude of unity without any units or dimensions. In Cartesian coordinates three unit vectors are adopted to specify the

positive directions of the three-axes. These are: **i** for the positive *x*-axis, **j** for the positive *y*-axis and **k** for the positive *z*-axis, as shown in **Figure 1.3**. This means that if a vector **A** is directed a long the positive *x*-axis with a magnitude of *A*, this vector can be written as **A**=*A***i**. Similarly, a vector **B,** directed a long the positive *y*-axis and has a magnitude B is written as **B**=*B***j**. The vector C=*C***k** means that it has a magnitude of *C* and directed along the positive *z*-axis. The minus sign in front of any vector indicates the opposite direction of that vector, i.e. **-i** refers to the negative *x*-axis, and so for the other two unit vectors.

1.5 COMPONENTS OF VECTORS

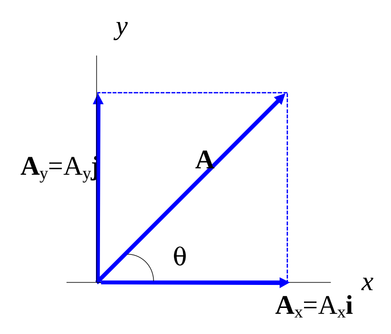


Figure 1.4 A vector **A** can be resolved into two perpendicular components, the *x*-component A_x and the *y*-component A_y.

Any vector **A** in a plane can be represented by the sum of two vectors, one parallel to the *x*-axis ($\mathbf{A}_x$), and the other parallel to the *y*-axis ($\mathbf{A}_y$) as shown in Figure 1.4, i.e.

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y . \tag{1.4}$$

Since, $\mathbf{A}_x = A_x\mathbf{i}$ and $\mathbf{A}_y = A_y\mathbf{j}$ we can write Equation 1.4 as

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} \qquad (1.5)$$

where A_x and A_y are the x-component and the y-component of the vector **A**, respectively. From **Figure 1.4** it is clear that

$$A_x = A\cos q \qquad \text{and} \qquad A_y = A\sin q \qquad (1.6)$$

The magnitude of the vector **A** is given by

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2} \qquad (1.7)$$

In general any vector **A** can be resolved into three components as

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + \mathrm{A}_z \mathbf{k} \qquad (1.8)$$

where A_z is called the z-component of the vector **A**. Note that if **A**=0, then A_x=0, A_y=0, and A_z=0.

Example 1.2 A vector **A** lying in the x-y plane has a magnitude A=50.0 units and is directed at an angle of 120° to the positive x-axis, as shown in **Figure 1.5**. What are the rectangular components of this vector?

Solution From Equation (1.6) we have

$$\mathrm{A}_x = A\cos q = 50\cos 120^{\circ} = -25.0\text{units}$$

and

$$\mathrm{A}_y = A\sin q = 50\sin 120^{\circ} = 43.3\text{units}$$

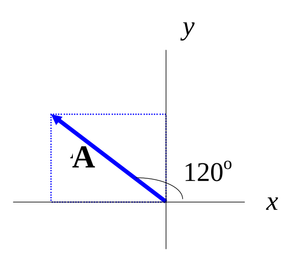


Figure 1.5

1.6 ADDING VECTORS

To add vectors analytically proceed as follow:

1) Resolve each vector into its components according to suitable coordinate axes.

2) Add, algebraically, the *x*-components of the individual vectors to obtain the *x*-component of the resultant vector. Do the same thing for the other components, i.e., if

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

and

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}\ ,$$

then the resultant vector **R** is

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k}$$

or

$$\mathbf{R} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k} \qquad (1.9)$$

Example 1.3 If $\mathbf{A} = 4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = -3\mathbf{i} + 7\mathbf{j}$, find the resultant vector **R**= **A+B.**

Solution From the given information we have

$$A_x = 4,\ A_y = 3,\ B_x = -3, \text{and}\quad B_y = 7$$

Now from Equation (1.9) we have

$$R_x = A_x + B_x = 1.00\ ,$$

and

$$R_y = A_y + B_y = 10.0$$

so we can write

$$\mathbf{R} = \mathbf{i} + 10\mathbf{j}$$

Example 1.4 A particle undergoes three consecutive displacements given by $\mathbf{d}_1 = (\mathbf{i} + 3\mathbf{j} - \mathbf{k})\,\text{cm}$, $\mathbf{d}_2 = (2\mathbf{i} - \mathbf{j} - 3\mathbf{k})\,\text{cm}$, and $\mathbf{d}_3 = (-\mathbf{i} + \mathbf{j})\,\text{cm}$. Find the resultant displacement of the particle.

Solution $\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3$, or

$$\begin{aligned}\mathbf{R} &= (1+2-1)\mathbf{i} + (3-1+1)\mathbf{j} + (-1-3+0)\mathbf{k} \\ &= (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})\,\text{cm}\end{aligned}$$

Example 1.5 A particle undergoes the following consecutive displacements: 4.3 m southeast, 2.4 m east, and 5.2 m north. Find the magnitude and the direction of the resultant vector.

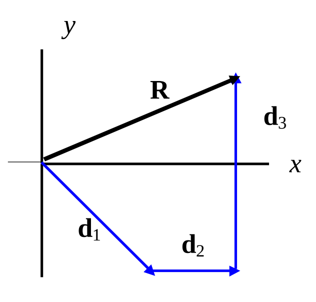


Figure 1.6

Solution If we denote the three displacements by $\mathbf{d}_1$, $\mathbf{d}_2$,and $\mathbf{d}_3$ respectively, we get the vector diagram shown in Figure 1.6. According to the coordinates system chosen, the three vectors can be written as

$$\begin{aligned}\mathbf{d}_1 &= (4.3\cos 45)\mathbf{i} - (4.3\sin 45)\mathbf{j} \\ &= (3.04\mathbf{i} - 3.04\mathbf{j})\,\text{m}\end{aligned}$$

$\mathbf{d}_2 = 2.4\mathbf{i}\,\text{m}$, and $\mathbf{d}_3 = 5.2\mathbf{j}\,\text{m}$

now,

$$\begin{aligned}\mathbf{R} &= \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 \\ &= (3.04 + 2.40 + 0)\mathbf{i} + (-3.04 + 0 + 5.20)\mathbf{j} \\ &= (5.44\mathbf{i} + 2.16\mathbf{j})\ \text{m}\end{aligned}$$

The magnitude of **R** is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(5.44)^2 + (2.16)^2} = 5.85\ \text{m}$$

To find the direction of a vector it is enough to determine the angle the vector makes with a specific axis. The angle q makes with the *x*-axis is

$$q = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2.16}{5.44}\right) = 21.66^{\circ}$$

Example 1.6 If $\mathbf{B} = 4\mathbf{i} - \mathbf{j}$, find the vector **A** such that $\mathbf{A} + \mathbf{B} = 5\mathbf{i}$.

Solution Since $\mathbf{A} + \mathbf{B} = 5\mathbf{i}$ then,

$$A_x + B_x = 5, \text{ and } A_y + B_y = 0$$

so we have

$$A_x = 5 - B_x = 5 - 4 = 1$$

and

$$A_y = -B_y = 1$$

This leads to

$$\mathbf{A} = \mathbf{i} + \mathbf{j}$$

1.7 PRODUCTS OF VECTORS

1- Multiplying a Vector by a Scalar: The product of a vector **A** and a scalar λ is a new vector with a direction similar to that of **A** if λ is positive but opposite to the direction of **A** if λ is negative. The magnitude of the new vector λ**A** is equal to the magnitude of **A** multiplied by the absolute value of λ, i.e.,

$$\lambda \mathbf{A} = \lambda A_x \mathbf{i} + \lambda A_y \mathbf{j} + \lambda A_z \mathbf{k} \qquad (1.10)$$

2- Scalar (Dot) Product: The scalar product of two vectors **A** and **B**, denoted by **A· B** , is a scalar quantity defined by,

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta \qquad (1.11)$$

with θ is the smallest angle between the two vectors. Since **i**, **j**, and **k** are unit vectors perpendicular to each other, and using Equation (1.11), we can write

$$\mathbf{i}\cdot\mathbf{j} = \mathbf{i}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{j} = 0 \qquad (1.12a)$$

$$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1 \qquad (1.12b)$$

Equations 1.12 imply that the scalar product of the vectors **A** and **B** can be written as

$$\mathbf{A}\cdot\mathbf{B} = A_x B_x + A_y B_y + A_z B_z \qquad (1.13)$$

Example 1.7 If $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, find the angle between the two vectors **A** and **B**

Solution From Equation (1.11) we have

$$\cos q = \frac{\mathbf{A}\cdot\mathbf{B}}{AB}.$$

But from Equation (1.7) we have

$$A = \sqrt{1+4+9} = 3.74, \text{ and}$$

$$B = \sqrt{4+9+4} = 4.12.$$

Using Equation (1.13) we find

$$\mathbf{A}\cdot\mathbf{B} = 2-6-6 = -10.0.$$

So we now have

$$\cos q = \frac{-10}{(3.74)(4.12)} = -0.69,$$

or

$$q = \cos^{-1}(-0.69) = 134^{o}$$

Example 1.8 Consider the two vectors A and B given in the previous example (example 1.7), find $2\mathbf{A}\cdot\mathbf{B}$

Solution $2\mathbf{A}=2\mathbf{i}-4\mathbf{j}+6\mathbf{k}$,
now from Equation 1.13, we obtain

$$(2\mathbf{A})\cdot\mathbf{B} = 4-12-12 = -20.0$$

Note that if one multiply the product $\mathbf{A}\cdot\mathbf{B}$ by 2, or multiply $\mathbf{A}\cdot(2\mathbf{B})$ the same result will be achieved, that is

$$2(\mathbf{A}\cdot\mathbf{B}) = (2\mathbf{A})\cdot\mathbf{B} = \mathbf{B}\cdot(2\mathbf{A})$$

3- Vector (cross) Product: The vector product of two vectors **A** and **B**, written as **A´B**, is a third vector **C** with a magnitude given by

$$C = |\mathbf{A}\times\mathbf{B}| = AB\sin q \qquad (1.14)$$

The vector **C** is perpendicular to the plane of **A** and **B**. Since there are two directions perpendicular to a given plane, the **right hand rule** is used to decide to which direction the vector **C**=**A´B** is directed. The rule states that the four fingers of the right hand are pointed along **A** and then curled toward B through the smaller angle between **A** and **B**. The thumb then gives the direction of **C**. From this rule one can verify that

A´B=-B´A (1.15)

and

i´i=**j´j**=**k´k**=0 (1.16a)
i´j=**k** (1.16b)
j´k=**i** (1.16c)
k×**i**=**j** (1.16d)

From Equations (1.16) we can write

$$\mathbf{A}\times\mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\times(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$
$$= (A_yB_z - A_zB_y)\mathbf{i} + (A_zB_x - A_xB_z)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

or, equivalently

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad (1.17)$$

Example 1.9 Find the vector product of the two vectors given in the previous example (example 1.7)

Solution

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 3 \\ 2 & 3 & -2 \end{vmatrix}$$

$$=(4-9)\mathbf{i} + (6+2)\mathbf{j} + (3+4)\mathbf{k} = -5\mathbf{i}+8\mathbf{j}+7\mathbf{k}$$

PROBLEMS

1.1 Show that the equation $v^2 = v_o^2 + 2ax$ is dimensionally correct, where *v* and v_o represent velocities, *x* is a distance, and *a* is an acceleration.

1.2 The period of a simple pendulum with the unit of time is given by $T = 2p\sqrt{\frac{l}{g}}$ where *l* is the length of pendulum, and *g* is the acceleration due to gravity. Show that the equation is dimensionally correct.

1.3 A vector **A** in the *x*-*y* plane is 12 units long and directed as shown in Figure 1.7, Determine the *x* and the *y* components of the vector.

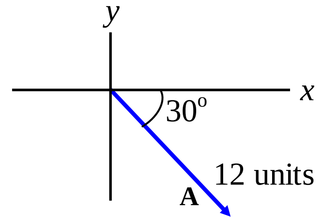


Figure 1.7 (Problem 1.3)

1.4 Vector **A** is 5 units in length and points along the positive *x*-axis. Vector **B** is 7 units in length and points along the negative *y*-axis.
a) Find **A**+**B**.
b) Find the magnitude and the direction of **A**+**B**

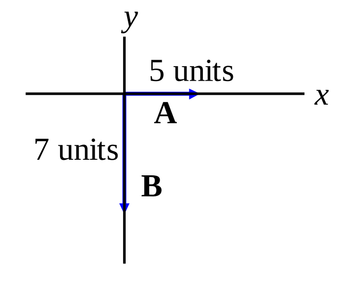
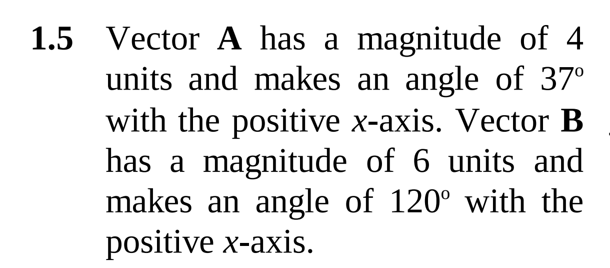


Figure 1.8 (Problem 1.4)



1.5 Vector **A** has a magnitude of 4 units and makes an angle of 37° with the positive *x*-axis. Vector **B** has a magnitude of 6 units and makes an angle of 120° with the positive *x*-axis.

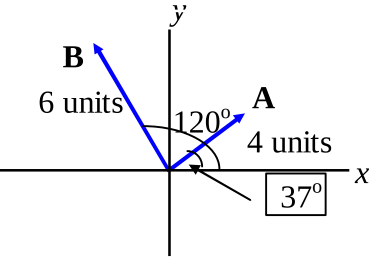


Figure 1.9 (Problem 1.5)

a) Find the x-component and the y-component of each vector.
b) Find **A-B**

1.6 A car is driven east for a distance of 30 km. It next driven north-east for a distance of 20 km, and then in a direction of 30^{o} north of west for a distance of 40 km. Draw the vector diagram of the three displacements and find their resultant (magnitude and direction).

1.7 Consider the three vectors shown in Figure 1.10. If A=8 m, B=5 m, and C=12 m, find
a) the x and the y -components of each vector,
b) the magnitude and the direction of the resultant vector.

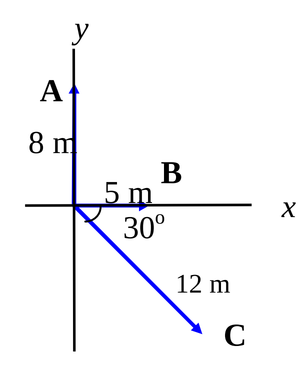


Figure 1.10 (Problem 1.7)

1.8 Find the magnitude and the direction of the resultant of the three displacement having components (-3,3) m, (2,4) m, and (5,-1) m.

1.9 Two vectors are given by **A**=3**i**+3**j**, and **B**=-2**i**+7**j**. Find
a) **A**+**B**,
b) **A**-**B**,
c) $|\mathbf{A}|$,
d) $|\mathrm{B}|$.

1.10 You are given the two vectors **A**=2**i**+3**j**, and **B**=6**i**+8**j**. Find the magnitudes and the directions of **A**, **B**, **A**+**B**, and **A**-**B**.

1.11 If **A**=2**i**+3**j**-**k**, and **B**=**i**+3**j**, and **C**=**i**+**j**+**k**, find

a) A+**B**+**C**
b) A-B-C

1.12 Given the two vectors **A**=4**i**-3**j**, and **B**=2**i**+4**j**. Find the vector **C** such that **A**+**B**+**C**=0.

1.13 Find a unit vector parallel to **A**=6**i**-8**j.**

1.14 Given the two vectors **A**=2**i**-3**j**+7**k**, and **B**=5**i**+**j**+2**k**, find
a) 2**A**+**B**
b) A-2**B**.

1.15 Find the scalar product **A.B** of the two vectors **A**, and **B** in problem 1.11.

1.16 Find the angle between the two vectors **A**=-2**i**+3**j**, and **B**=3**i**-4**j**.

1.17 For what values of λ are the two vectors **A**=**i**+2**j**+λ**k**, and **B**=2λ**i**-2**j**+2λ**k** perpendicular to each other.

1.18 If $\mathbf{A} = 2\mathbf{i} + 6\mathbf{j}, \mathbf{A} - \mathbf{B} = \mathbf{C},$ and $\mathbf{A} + \mathbf{B} = 2\mathbf{C}$ find **B** and **C**.

1.19 Using the scalar product, prove the law of cosine (the law is stated in appendix D).

1.20 Given **A**=2**i**+**j**+**k**, and **B**=**i**+3**j**. Find
a) A´B
b) A· (**A´B**)

1.21 Three vectors are given by **A**=**i**+**j**+**k**, **B**=**i**+2**j**, and **C**=2**i**-**k**. Find
a) A· (**B**+**C**)
b) A· (**B´C**)

c) $\mathbf{A}\times(\mathbf{B}+\mathbf{C})$

1.22 The two vectors **A**, and **B** represent concurrent sides of a parallelogram as shown in Figure 1.11. Show that the area of the parallelogram is $|\mathrm{A}\times\mathrm{B}|$.

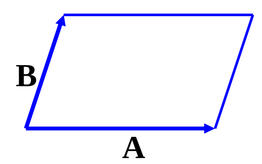


Figure 1.11 Problems 1.22

1.23 Find a unit vector perpendicular to the plane of the two vectors given in problem 1.17.

1.24 Two vectors of magnitudes *A* and *B* make an angle θ with each other when placed tail to tail (see Figure 1.12). Prove that the magnitude of their sum, *R* is given by

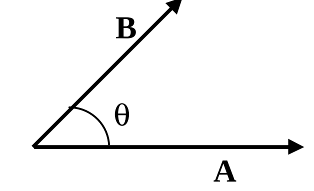


Figure 1.12 Problems 1.24

$$R = \sqrt{A^2 + B^2 + 2AB\cos q}$$

1.25 Prove that for any three vectors **A**, **B**, and **C**

A· (**B**´ **C**)=(**A**´ **B**)· **C**

CHAPTER 2

LINEAR MOTION

Motion of an object is the continuous change in the position of that object. In this chapter we shall consider the motion of a particle in a straight line, which will be taken to be one of the coordinate axes.

2.1 AVERAGE VELOCITY

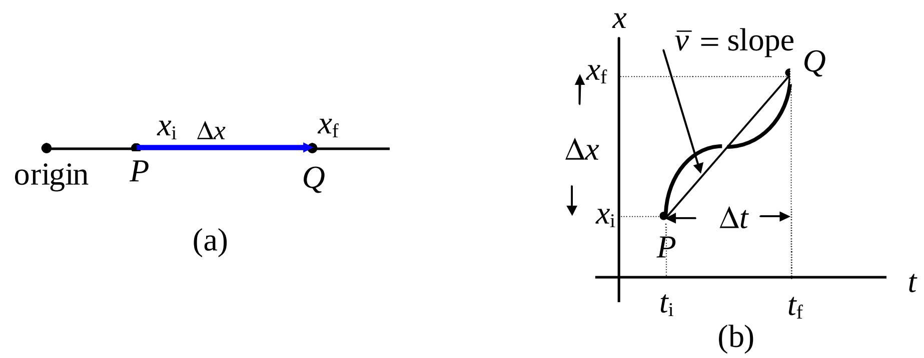


Figure 2.1 (a) A particle moves along the x-axis. It starts at point P at $t=t_i$ and finishes at point Q at a later time $t=t_f$. (b) The position-time graph for the same particle. The average velocity is the slope of the straight line connecting the initial and the final points.

Consider a particle moving along a straight line (x-axis) as in Figure 2.1(a). It starts at point P, with position x_i, at time t_i, and finishes at point Q, with position x_f, at time t_f. In the time interval $\Delta t = t_f - t_i$ the displacement of the particle is $\Delta x = x_f - x_i$. The average velocity, $\bar{v}$, of the particle during the time interval Δt is now defined as the ratio of Δx to Δt, or

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad (2.1)$$

From this relation, it is clear that the velocity has a dimension of length divided by time (L/T) with a unit of m/s, cm/s, and ft/s according to the SI, Gaussian, and British system, respectively. The average velocity, as it is clear from Equation 2.1, depends only on the initial and the final positions of the particle. This means that if a

particle starts from a point and return back to the same point, its displacement, and so its average velocity is zero. Fig 2.1(b) shows the variation of x with t where the average velocity is given by the slope of the straight line between points P and Q.

Remark: There is a difference between distance and displacement. **Distance**, a scalar quantity, is the actual long of the path traveled by a particle, but **displacement**, a vector quantity, is the shortest distance between the initial and the final positions of the particle.

Speed is the magnitude of the velocity. This means that the speed can never be negative. **The average speed** differs from average velocity in that it covers the total distance rather than the total displacement, that is

$$\text{average speed} = \frac{\text{total distance}}{\Delta t} \tag{2.2}$$

Although the velocity and the displacement are vectors, we do not need to write them as vectors since all vectors of this chapter have only one component.

2.2 INSTANTANEOUS VELOCITY

The instantaneous velocity, v, is defined as the value of $\bar{v}$ when Δt approaches zero (becomes instant), that is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.3}$$

In the position-time graph (Figure 2.2), v at some instant is the slope of the tangent at that instant.

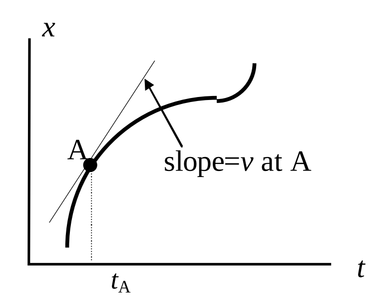


Figure 2.2 Position time graph for a particle moving along the x-axis. The instantaneous velocity v at point A is defined as the slope of the tangent line at t_A .

Example 2.1 The position of a particle varies with time according to $x = t^2 + 3t$ with x in m and t in s.

a) Find $\bar{v}$ for the interval t=0 to t=2 s
b) Find v at t=1.5 s

Solution a) to find x_i and x_f , we have to substitute for t_i and t_f in the x-t relation, that is,

$$x_i = (t_i)^2 + 3(t_i) = 0\,,$$

and

$$x_f = (t_f)^2 + 3(t_f) = 4 + 6 = 10\text{m}\,.$$

Now, from Equation 2.1, we have

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{10 - 0}{2 - 0} = 5 \ \text{m/s.}$$

b) From Equation 2.3, we get

$$v = \frac{dx}{dt} = 2t + 3\,.$$

The instantaneous velocity at t= 1.5 s is obtained by substituting for t= 1.5 s in the last equation

$$v = 2(1.5) + 3 = 6 \text{ m/s}$$

2.3 ACCELERATION

When the velocity of a moving body changes with time, we say that the body has acceleration. The **average acceleration** $\bar{a}$, during a time interval Δt, is the ratio of the change in velocity, Δv, to the time interval Δt, i.e.,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_\mathrm{f} - v_\mathrm{i}}{t_\mathrm{f} - t_\mathrm{i}} \tag{2.4}$$

The unit of the acceleration is m/s^2 in SI system.

In analog with the instantaneous velocity, the **instantaneous acceleration**, a, is defined as the limit of the average acceleration as Δt approaches zero, i.e.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \tag{2.5}$$

From Equation 2.5, it is clear that the acceleration is the slope of the velocity-time graph. As $v = \frac{dx}{dt}$ (from Equation 2.3), then Equation 2.5 can be rewritten as

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{2.6}$$

Example 2.2 The velocity of an object moving along the *x*-axis varies with time according to the relation v=5t-3 with v in m/s and t in s.
a) Find $\bar{a}$ during the interval t=1 s to t=2 s
b)Find a at t=2s

Solution
a) $v_i = 5(t_i) - 3 = 5(1) - 3 = 2$ m/s,

and

$$v_f = 5(t_f) \; - 3 = 5(2) - 3 = 7 \text{ m/s}.$$

So, from Equation 2.4

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{7-2}{2-1} = 5 \text{ m/s}^2.$$

b) Using Equation 2.5, we obtain

$$a = \frac{dv}{dt} = 5 \text{ m/s}^2.$$

Example 2.3 The position-time graph of a particle moving along the *x*-axis is given in Figure 2.3. Find
a) $\bar{v}$ during the interval t=2 s to t=5 s.
b)$\bar{a}$ during the interval t=0.5 s to t=2.5 s

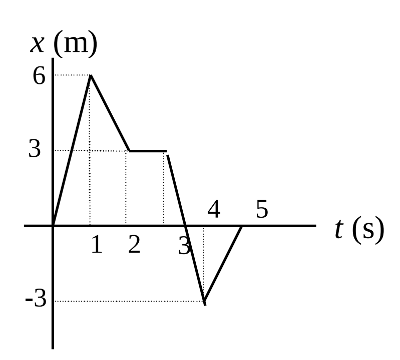


Figure 2.3 Example 2.3

Solution a) As it is clear from the graph, at t = 2 s $x_i = 3$ m, and at t = 5 s

$x_f = 0$. Now from Equation 2.1, we get

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{0-3}{5-2} = -1\text{m/s}$$

b) Since *v* at any point is the slope of the *x*-*t* graph at that point, we have at *t*=0.5 s *v*=6m/s, and at t=2.5 s *v*=0, so

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{0-6}{2.5-0.5} = -3 \text{ m/s}^2$$

٢.٤ LINEAR MOTION WITH CONSTANT ACCELERATION

The simplest type of linear motion is the uniform motion in which the acceleration is constant. In such a case $\bar{a}$ =*a*, so from Equation 2.4, we obtain

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{v - v_o}{t} \tag{2.7}$$

where we denote v_i by v_o, v_f by *v*, and t_f by *t*, and take t_i=*0*. The above equation now is

$$\boxed{v = v_0 + at} \tag{2.8}$$

Also, because *a* is constant, we can write

$$\bar{v} = \frac{v + v_o}{2} \tag{2.9}$$

Using Equations 2.1 and 2.9, we obtain

$$\frac{x - x_o}{t} = \frac{v + v_o}{2} \tag{2.10}$$

Substituting for v from Equation 2.8 and rearrange, we obtain

$$x - x_o = v_o t + \tfrac{1}{2} a t^2 \tag{2.11}$$

Now substituting for t from Equation 2.8 into Equation 2.10 we get

$$v^2 = v_o{}^2 + 2a(x - x_o) \tag{2.12}$$

When the velocity is constant, (a=0) it is clear from Equations 2.8 and 2.11 that

$$v = v_o \text{ and } x - x_o = vt \tag{2.13}$$

For simplicity, the origin of the coordinates is often chosen to be coincident with x_o (i.e. x_o=0). Equations 2.8, 2.11, and 2.12 are the fundamental three equations that govern the linear motion with constant acceleration.

Strategy for solving problems with constant acceleration:

(i) Choose your coordinates such that the particle begins its motion from the origin (x_o =0).
(ii) Decide the sense of the positive direction.
(iii) Make a list of the known quantities. Do not forget to write any vector quantity (x, v, v_o, a) that have a direction opposite to your positive sense as a negative quantity.
(iv) Make sure that all the quantities have the same system of units.

(v) According to what is given and what is requested, you can easily decide which equation or equations from equations 2.8, 2.11, and 2.12 you need to solve for the unknowns.

Example 2.4 A car starts from rest and moves with constant acceleration. After 12 s its velocity becomes 120 m/*s*. Find,
a) the acceleration of the car
b) the distance the car travels in the 12 s

Solution Let the direction of motion be along the positive *x*-axis, where the car starts from the origin at *t*=0. Now
v_o=0, v=120 m/s, t=12 s.

a) Using Equation 2.8, namely $v = v_0 + at$, we have

$$a = \frac{v - v_o}{t} = \frac{120 - 0}{12} = 10\,\text{m/s}^2 .$$

b) Equation 2.11 reads $x = v_0 t + \frac{1}{2} a t^2$, substituting for v_o, t, and a yields

$$x = 0 + (0.5)(10)(12)^2 = 720\,\text{m} .$$

2.5 FREELY FALLING BODIES

A freely falling body is any body moving freely under the influence of gravity regardless of its initial motion. Neglecting the air resistance and assuming that the gravitational acceleration, denoted by *g*, is constant, we can consider the motion of a freely body as a linear motion with constant acceleration. Taking your axis to be the *y*-axis with the positive sense upward, Equations 2.8, 2.11, and 2.12 will apply with the substitutions, $x \rightarrow y$ and $a \rightarrow -g$, i.e.,

$$v = v_0 - gt \qquad (2.14)$$

$$y - y_0 = v_0 t - \tfrac{1}{2} g t^2 \qquad (2.15)$$

$$v^2 = v_o^2 - 2g(y - y_o) \qquad (2.16)$$

The first substitution is because the motion is now vertical and the negative sign in the second substitution indicates that the acceleration is downward.

Remark: Remember that the gravitational acceleration, g, is constant in magnitude and in direction and this means that the negative sign of g in Equations 2.14-2.16 will not be changed unless you change your positive sense, regardless of the direction of motion.

Example 2.5 An object is thrown vertically upward with an initial speed of 25 m/s.
a) How long does it take to reach its maximum height?
b) What is the maximum height?
c) How long does it take to return to the ground?
d) What is its velocity just before striking the ground?

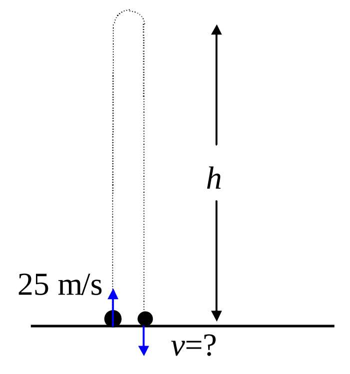


Figure 2.5 Example 2.5.

Solution The track of the object is shown in Figure 2.5.
a) At the maximum point v=0. From Equation 2.14 we have

$$t = \frac{v_o - v}{g} = \frac{25 - 0}{9.8} = 2.55s\,.$$

b) Using Equation 2.16, i.e. $v^2 = v_o^2 - 2gy$ we have

$$h = y = \frac{v_o^2 - v^2}{2g} = \frac{(25)^2 - 0}{2(9.8)} = 31.9\text{m}$$

c) When returning to the ground, the displacement of the object is zero (y=0). So from Equation 2.15, we have

$$0 = v_o t - \tfrac{1}{2} g t^2\,.$$

Solving for t we get

$$t = \frac{2v_o}{g} = \frac{25}{4.9} = 5.1\text{s}\,.$$

d) From Equation (2.14) we have

$$v = v_0 - gt = 25 - 9.8(5.1) = -25\text{m/s}$$

The minus sign indicates that **v** is directed downward.

Example 2.6 A student, stands at the edge of the roof of a building, throws a ball vertically upward with an initial speed of 20 m/s. The building is 50 m high, and the ball just missed the edge of the roof in its way down, (Figure 2.6). Find,

a) the time needed for the ball to return to the level of the roof

b) the velocity and the position of the ball at t =5 s

c) the velocity of he ball just before hitting the ground

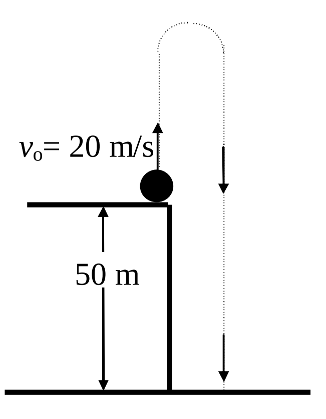


Figure 2.6 Example 2.6.

Solution

a) When the ball returns to the level of the roof, its displacement, y, is zero. Substituting in Equation 2.15 we get

$$0 = 20t - \tfrac{1}{2}(9.8)t^2 .$$

Now solving for t we get

$$t = \frac{40}{9.8} = 4.08\text{s} .$$

b) Using Equation 2.14, $v = v_0 - gt$, we obtain

$$v = 20 - 9.8(5) = -29 \text{ m/s} ,$$

and using Equation 2.15, $y = v_o t - \tfrac{1}{2} g t^2$, we have

$$y = 20(5) - 4.9(25) = -22.5 \text{ m} .$$

c) From the equation $v^2 = v_o^2 - 2gy$, (Equation 2.16), we obtain

$$\begin{aligned} v^2 &= (20)^2 - 2(9.8)(-50) \\ &= 400 + 980 = 1380 \text{ m}^2/\text{s}^2 , \end{aligned}$$

from which we find

$$v = -37.15 \text{ m/s}$$

The positive solution is rejected because the ball hits the ground while falling.

٢.٦ RELATIVE MOTION

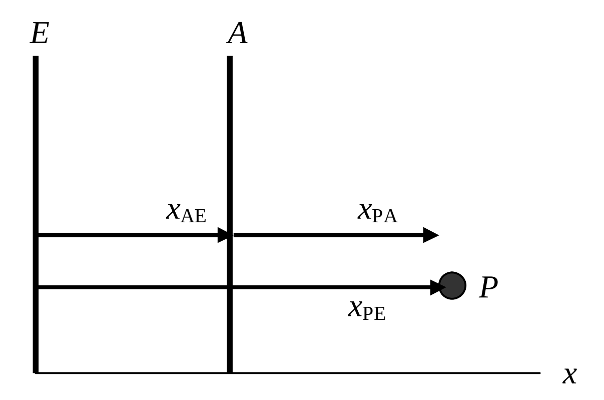


Figure 2.7 Omer (frame *E*) and Ahmed (frame *A*) observe particle *P*. Omer is stationary while Ahmed is moving with constant speed relative to Omer.

The displacement, velocity, and acceleration of a particle are always described with respect to a specific coordinate system or frame of reference. Usually, this frame of reference is taken to be fixed, but what happen if this frame of reference is in motion with respect to another frame of reference.

Suppose that two persons want to observe the motion of a particle *P*. The first person Omer is stationary with respect to the earth, while the second person, Ahmed is moving with constant speed relative to the earth. Let the reference of frame of Omer be referred by frame *E* (frame of the earth) and the reference of frame of Ahmed by frame *A*. The displacement of the particle with respect to the earth (Omer) is denoted by x_{PE}, and x_{pA} is the displacement of the particle with respect to Ahmed. From Figure 2.7 we can write

$$x_{PE} = x_{pA} + x_{AE} \qquad 2.17$$

where x_{AE} refers to the displacement of Ahmed with respect to the earth.
If we differentiate Equation 2.17 with respect to time, we get

$$v_{PE} = v_{pA} + v_{AE} \qquad (2.18)$$

In the last equation v_{PE} is the velocity of *P* relative to *E*, v_{PA} is the velocity of *P* relative to *A*, and v_{AE} is the velocity of *A* relative to *E*.

Example 2.7 A man, in a car, is driving on a straight highway at constant speed of 70 km/h relative to the earth. Suddenly, he spots a truck traveling in the same direction with constant speed of 60 km/h relative to the earth.
a) What is the velocity of the truck relative to the car?
b) What is the velocity of the car relative to the truck?
c) If the car spots the truck when they are 1.5 km apart, how long does it take the car to overtake the truck?

Solution
a) Let v_{TC} denotes the velocity of the truck relative to the car, and v_{TE} and v_{CE} denote, respectively, the velocities of the truck and the car relative to the earth. Now from Equation 2.18 we have

$$v_{TE} = v_{TC} + v_{CE}$$

or

$$v_{TC} = v_{TE} - v_{CE} = 60 - 70 = -10\,\text{km/h}$$

The minus sign indicates that the car is moving in the opposite direction relative to the car.
b) Again from Equation 2.18, the velocity of the car relative to the truck v_{CT} is

$$v_{CE} = v_{CT} + v_{TE}$$

or

$$v_{CT} = v_{CE} - v_{TE} = 70 - 60 = 10\,\text{km/h}$$

Note that $v_{CT} = -v_{TC}$ as one should expect.

c) The time needed for the car to overpass the truck is

$$t = \frac{1.5}{v_{CT}} = \frac{1.5}{10} = 0.15\,\text{h} = 9\,\text{minutes}$$

PROBLEMS

٢.١ An automobile travels on a straight track at 80 km/h for 30 minutes. It then travels at 60 km/h for another 20 minutes.

a) What is the average velocity of the automobile during the 50 minutes trip?

b) What is the average speed of the automobile during the whole trip.

2.2 A particle moves according to the equation $x = 8t^2 + 2$, where x in meters and t in seconds.

a) Find the average velocity during the time interval from 2 s to 4 s.

b) Find the instantaneous velocity at t=2.5 s.

2.3 Figure 2.8 is a plot of the displacement $x(t)$ for a particle that is initially at rest, then moves forward, and then stops.

a) Find the average velocity of the particle during the interval t=1s to t=10 s.

b) Find the instantaneous velocity at t=4 s.

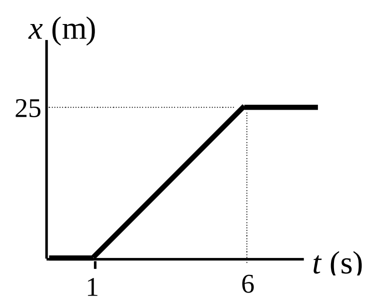


Figure 2.8 Problem 2.3

2.4 A particle moves along the x-axis according to $x = 5t^2 + 30t$, where x in meters and t in seconds. Calculate

a) the instantaneous velocity at t=3.0 s,

b) the average acceleration during the first 3.0 s,

c) The instantaneous acceleration at t=3.0 s.

2.5 A car traveling initially at a speed of 80 km/h is accelerated uniformly to a speed of 105 m/s in 15 s. What is the distance traveled by the car during this 15 s interval.

2.6 The initial speed of a body is 6.4 m/s. If its acceleration is 3.5 m/s^2, find its speed after 2.8 s.

2.7 An automobile moving with constant acceleration travels a distance of 60 m in 6 s. If its final speed is 15 m/s,

a) what is its acceleration?

b) what is its initial speed?

2.8 A train starts from rest at a station and accelerates at a rate of 600 km/h^2 for 10 minutes. After that it travels at constant speed for 1 h, and then slows down at -800 km/h^2 until it stops at another station. Find the total distance covered by the train.

2.9 A car traveling at constant speed of 50 km/hr is 60 m from a stationary lorry when the driver slams on the brakes. If the car just missed the impact,

a) find the acceleration of the car,

b) find the time interval from the instant the driver slams on the brakes to the instant the car stopped.

2.10 At the instant the traffic light turns green, a car starts from rest with constant acceleration of 2 m/s^2. At the same instant a truck, traveling with constant speed of 10 m/s overtakes and passes the car.

a) How far beyond the traffic light will the car overtake the truck?.

b) What is the speed of the car at that instant.

2.11 A car traveling at a constant speed of 60 km/h passes a trooper hidden behind a tree. Two second after the car passes the tree, the trooper sets in chase after the car with constant

acceleration of 3.5 m/s^2. How long does it take the trooper to overtake the car?

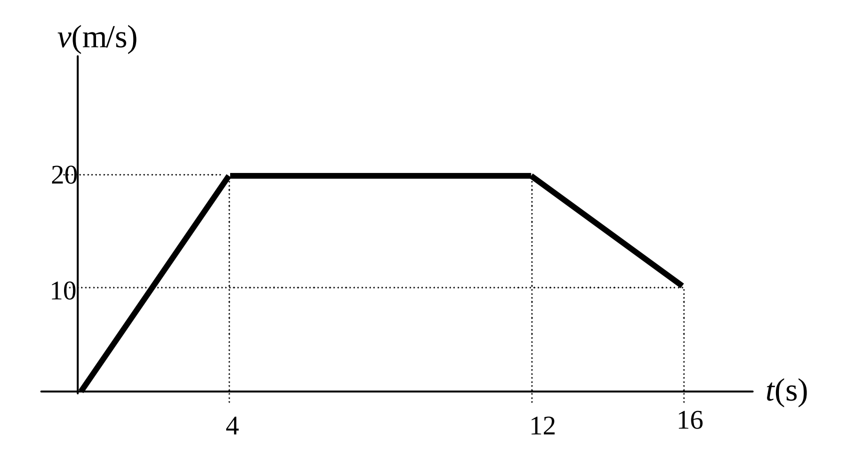


Figure 2.9 problem 2.12

2.12 The graph in Figure 2.9 depicts the motion of an automobile in a street.

a) Find the displacement of the automobile during the following intervals: 0-4s, 4s-12s, and 12s-16s.

b) Find the velocity of the automobile at t=2s, t=8s, and at t=14s.

c) Find the acceleration of the automobile during 0-4s, 4s-12s, and 12s-16s.

2.13 A small ball is thrown vertically upward and rises to a maximum height of 20 m.

a) With what velocity the ball is thrown.

b) How long will it be in the air.

2.14 A small ball is released from a height of 45 m.

a) Find the velocity of the ball just before it hits the ground.

b) How long does it take the ball to hit the ground?

2.15 A body is thrown vertically upward from the edge of a building 21m high. In its way down it misses the building and hits the ground below with a speed of 24 m/s, as in Figure 2.10.

a) With what velocity the body is thrown.

b) What is the time of flight?

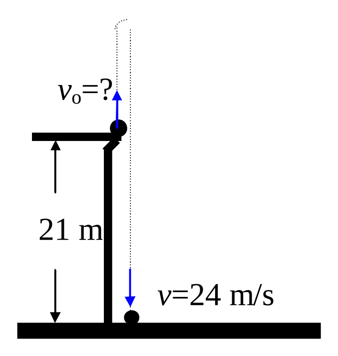


Figure 2.10problem .15

2.16 Refer again to Figure 2.10. Now the height of the building is 30m and the body is thrown with a speed of 16 m/s.

a) What is the average velocity of the body during the whole flight?

b) What is the average speed of the body during the same interval?

2.17A bird is traveling vertically upward at a constant speed of 22.5m/s with a piece of bread in his mouth. When it is 150 m above the ground the bread piece is released from the bird's mouth.

a) Find the speed with which the bread hits the ground.

b) How long does the bread take to reach the ground?

c) What is the average speed of the bread?

d)What is the average velocity o the bread?

2.18 A falling object starting from rest requires 1.5 s to travel the last 30m before hitting the ground. From what height above the ground did it fall.

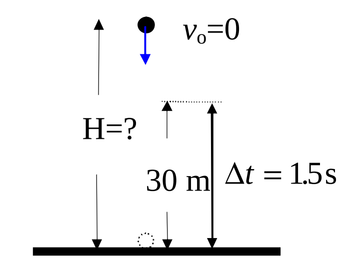


Figure 2.11 (Problem 2.18)

2.19 A ball is thrown vertically upward. In its way up it passes a point P with speed *v*, and another point Q, 2.5 m higher than P with a speed 0.25*v*. Find the value of *v*.

٢.٢٠ A child releases a ball from the balkone of his house that is 27 m high. The ball hits a man walking with constant speed. The man was 20 m from the point of impact when the ball was released. What was the speed of the walking man?

٢.٢١ Two objects released from the same height 0.5 s apart. How long after the first object begins to fall will the two objects be 8.0 m apart?

2.22 A parachutist jumps horizontally to the air from a height of 2000 m and falls freely 400m before the parachute opens. The parachutist hits the ground with a speed of 5 m/s.

a) Calculate the acceleration of the parachute while it is open.

b) How long was the parachutist in the air.

2.23 A student in his way back to the home is traveling at constant speed of 50 km/h. What would be the velocity of the trees in the roadside as observed by the student?

2.24 A river has a steady speed of 0.2 m/s. A man swims upstream a distance of 500 m and returns to the starting point. If the man can swim at a speed of 1.0 m/s in still water, how long does the trip take?

2.25 A parachutist throws a stone vertically upward with a speed of 12 m/s relative to himself. At the instant he throws the stone he is 120 m high and falling at 4m/s. How long does it take the stone to reach the ground?

CHAPTER 3

PLANAR MOTION

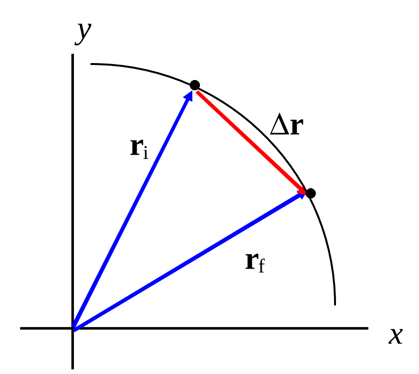


Figure 3.1 The position vector of a particle moving in the *x*-*y* plane.

In chapter 2 we have studied the motion that takes place along a straight line. In this chapter we extended our study to the motion that takes place along a curve, that is the motion in a plane.

3.1 DISPLACEMENT, VELOCITY, AND ACCELERATION.

Consider a particle moves along the curve shown in Figure 3.1. The initial position vector of the particle at $t=t_i$ is $\mathbf{r}_i$ and at later time t_f its final position vector is $\mathbf{r}_f$. As $\mathbf{r}_i$ and $\mathbf{r}_f$ are two-dimensional vectors, we can write

$$\mathbf{r}_i = x_i\mathbf{i} + y_i\mathbf{j}, \text{and} \mathbf{r}_f = x_f\mathbf{i} + y_f\mathbf{j} \tag{3.1}$$

Now, the displacement vector $\Delta\mathbf{r}$ is

$$\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i = (x_f - x_i)\mathbf{i} + (y_f - y_i)\mathbf{j},$$

or

$$\Delta\mathbf{r} = \Delta x\mathbf{i} + \Delta y\mathbf{j}, \tag{3.2}$$

where $\Delta x = x_f - x_i$, and $\Delta y = y_f - y_i$.

The **average velocity** of a particle moving in a plane is

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\mathbf{i} + \frac{\Delta y}{\Delta t}\mathbf{j} = \bar{v}_x\mathbf{i} + \bar{v}_y\mathbf{j}, \quad (3.3)$$

and the **instantaneous velocity** is now given as

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} = v_x\mathbf{i} + v_y\mathbf{j}. \quad (3.4)$$

Similarly, the average and the instantaneous acceleration for the case of a planer motion is

$$\bar{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\mathbf{i} + \frac{\Delta v_y}{\Delta t}\mathbf{j} = \bar{a}_x\mathbf{i} + \bar{a}_y\mathbf{j}. \quad (3.5)$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{dv_x}{dt}\mathbf{i} + \frac{dv_y}{dt}\mathbf{j} = a_x\mathbf{i} + a_y\mathbf{j}. \quad (3.6)$$

Example 3.1 The coordinates of a particle moving in the *x*-*y* plane are given as a function of time by $\mathbf{r} = 2t\mathbf{i}+(19-2t^2)\mathbf{j}$ where **r** in m and *t* in s.
a) What is the average velocity of the particle during the interval t = 0 to t = 2 s.
b) What is the velocity of the particle at t = 2 s?
c) What is the acceleration of the particle at t = 2 s?
Solution a) As $t_i = 0$, we get $\mathbf{r}_i = 19\mathbf{j}\text{m}$, from which we find that $x_i = 0$, and $y_i = 19\text{m}$. Similarly, as t_f = 2 s, we obtain $\mathbf{r}_f = (4\mathbf{i} + 11\mathbf{j})$ m, so $x_f = 4\text{m}$, and $y_f = 11\,\text{m}$. Now

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i} = \frac{4-0}{2} = 2 \text{ m/s}$$
$$\bar{v}_y = \frac{y_f - y_i}{t_f - t_i} = \frac{11-19}{2} = -4 \text{ m/s},$$

so we write

$$\bar{\mathbf{v}} = (2\mathbf{i} - 4\mathbf{j}) \text{ m/s}.$$

b)Using Equation (3.4) we have

$$v_x = \frac{dx}{dt} = 2 \text{ m/s, and}$$
$$v_y = \frac{dy}{dt} = -4t.$$

For t=2 s we have $v_y = -8 \text{m/s}$, and

$$\mathbf{v} = (2\mathbf{i} - 8\mathbf{j}) \text{m/s}.$$

c)Using Equation (3.6) we obtain

$$a_x = \frac{dv_x}{dt} = 0 \text{ , and} \quad a_y = \frac{dv_y}{dt} = -4 \text{ m/s}^2 \text{ ,}$$

or

$$\mathbf{a} = -4\mathbf{j} \text{ m/s}^2 \text{ .}$$

It is clear now that the planar motion can be considered as a vector sum of two-perpendicular linear motions. Parallel to this line, if **a** is constant, then a_x and a_y are consequently constants, then by the laws derived in chapter 2 we can write, by letting $x_o = y_o = 0$,

$$v_x = v_{ox} + a_x t \qquad v_y = v_{oy} + a_y t \qquad (3.7)$$

$$x = v_{ox} t + \tfrac{1}{2} a_x t^2 \qquad y = v_{oy} t + \tfrac{1}{2} a_y t^2 \qquad (3.8)$$

$$v_x^2 = v_{ox}^2 + 2a_x x \qquad v_y^2 = v_{oxy}^2 + 2a_y y \qquad (3.9)$$

3.2 PROJECTILE MOTION

As an example of planar motion, we will consider the motion of a particle given an initial velocity and then follows a path determined by gravity with the assumption that the air effect is neglected. Such a motion is called the **projectile motion** and the path followed by the projectile is called its **trajectory**, (see Figure 3.2) .

As the acceleration of gravity is always toward the center of the earth (straight downward) with a magnitude of g, the acceleration of a projectile is always given as

$$\mathbf{a} = -g\mathbf{j}, \text{ that is}$$
$$a_x = 0, \text{ and } a_y = -g . \qquad (3.10)$$

This means that the projectile motion can be considered as a vector sum of a vertical motion with a_y = $-g$ (the positive sense is taken upward) and a horizontal motion with a_x = 0 (constant velocity). The formulas govern the motion of a projectile is then

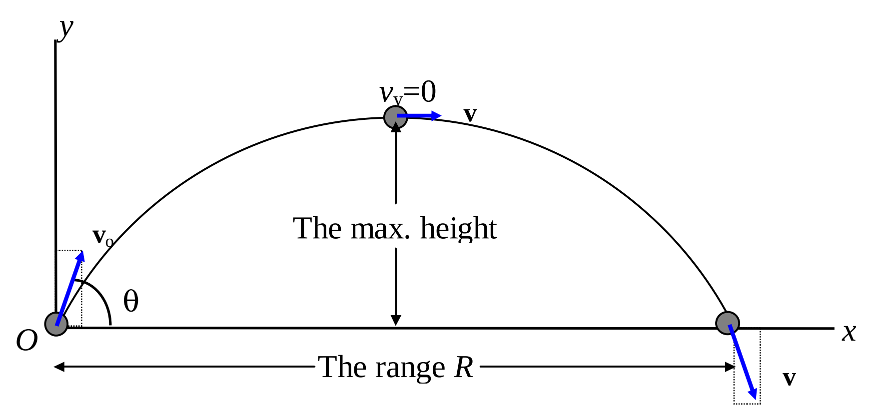


Figure 3.2 The trajectory of a projectile that is launched from the origin with an initial velocity $\mathbf{v}_o$. The total horizontal distance is called the range of the projectile.

$$v_x = v_{ox} = \frac{x}{t}. \tag{3.11}$$

$$v_y = v_{oy} - gt. \tag{3.12}$$

$$y = v_{oy}t - \tfrac{1}{2}gt^2. \tag{3.13}$$

$$v_y^2 = v_{oy}^2 - 2gy. \tag{3.14}$$

If one eliminate t from Equations 3.11 and 3.13 he obtain

$$y = (v_o \tan\theta)x - \left(\frac{g}{2v_o^2\cos^2\theta}\right)x^2 \tag{3.15}$$

This equation is the equation of the parabola, so the path of a projectile is parabolic.

Example 3.2 A projectile is fired from point O with initial speed v_o that make an angle θ with the horizontal as shown in Fig 3.2.

a) Find the maximum height h of the projectile.
b)Find the horizontal range R of the projectile.
Solution Resolving the initial velocity v into its components we get

$$v_{\mathrm{ox}} = v_{\mathrm{o}} \cos q \text{ , and } v_{\mathrm{oy}} = v_{\mathrm{o}} \sin q .$$

a)At the maximum height (h), $v_{\mathrm{y}} = 0$, using the equation

$$v_{\mathrm{y}}^2 = v_{\mathrm{oy}}^2 - 2gy,$$
$$0 = (v_{\mathrm{o}} \sin q)^2 - 2gh.$$

we obtain

$$h = \frac{v_0^2 \sin^2 q}{2g} .$$

b) To find the range we need the time of flight. Noting that for the total time of flight y=0, so we can use $y = v_{\mathrm{oy}} t - \frac{1}{2} g t^2$ to obtain

$$t = \frac{2 v_{\mathrm{oy}}}{g} = \frac{2 v_{\mathrm{o}} \sin q}{g} .$$

Now substituting in the equation $x = v_{\mathrm{ox}} t$ we have

$$x = v_{\mathrm{o}} \cos q \left(\frac{2 v_{\mathrm{o}} \sin q}{g}\right) ,$$

so

$$R = x = \frac{v_{\mathrm{o}}^2 \sin 2q}{g} .$$

Example 3.3 A ball is thrown from the top of a building 30 m in height. If the ball is thrown upward with a speed of 25 m/s that making an angle of 37^{o} with the horizontal.
a)What is the time of flight?
b)Where does the ball hit the ground?
c)What is the speed of the ball just before it hits the ground?

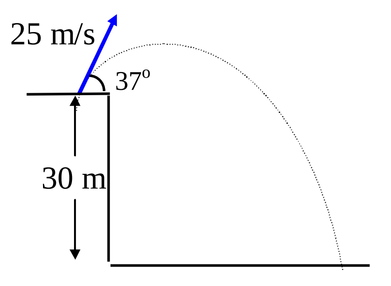


Figure 3.3 Example 3.3

Solution Resolving the initial velocity v_o, as

$$v_{ox} = v_x = v_o \cos q = 25\cos 37 = 20\text{m/s}.$$
$$v_{oy} = v_o \sin q = 25\sin 37 = 15\text{m/s}.$$
$$y = -30\text{m}.$$

a) To find t we can use $y = v_{oy}t - \frac{1}{2}gt^2$, so

$$-30 = 15t - 4.9t^2.$$

Solving for t we get t=4.44 s.

b) $x = v_x t = (20)(4.44)$
$= 88.8\,\text{m}$ from the base of the building.

c) $v_y = v_{oy} - gt = 15 - 9.8(4.44)$
$= -28.5\text{m/s}.$

Since v_x is constant, we can write

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = 20\mathbf{i} - 28.5\mathbf{j}\text{m/s},$$

and for the speed

$$v = \sqrt{v_x^2 + v_y^2} = 34.8\,\text{m}/s\,.$$

Example 3.4 A ball rolls off the edge of a tabletop 1.2m above the floor with an initial speed of 3 m/s.
a)Where does the ball strike the floor?
b)What is the final velocity of the ball?

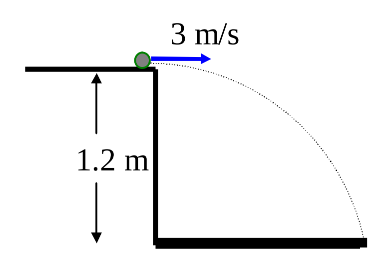


Figure 3.3 Example 3.3

Solution $v_{ox} = v_x = 3\text{m}/\text{s}$, $v_{oy} = 0,$ and $y = -1.2\text{m}$

a) To determine the point, at which the ball strikes the floor, it is enough to find the horizontal displacement of the ball. To find it we need the time of flight that can be found using the equation,

$$y = v_{oy}t - \tfrac{1}{2}gt^2\text{,so}$$
$$-1.2 = 0 - 4.9t^2,$$

from which we find $t = \sqrt{\dfrac{1.2}{4.9}} = 0.49\text{s}\,.$

Now

$$x = v_x t = 3(0.5) = 1.48\text{m}\,.$$

b) The *y*-component of the velocity is

$$v_y = v_{oy} - gt = 0 - 9.8(0.5) = -4.8\text{m}/\text{s}\,.$$

Now the velocity is

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = 3\mathbf{i} - 4.8\mathbf{j}\,\mathrm{m/s},$$

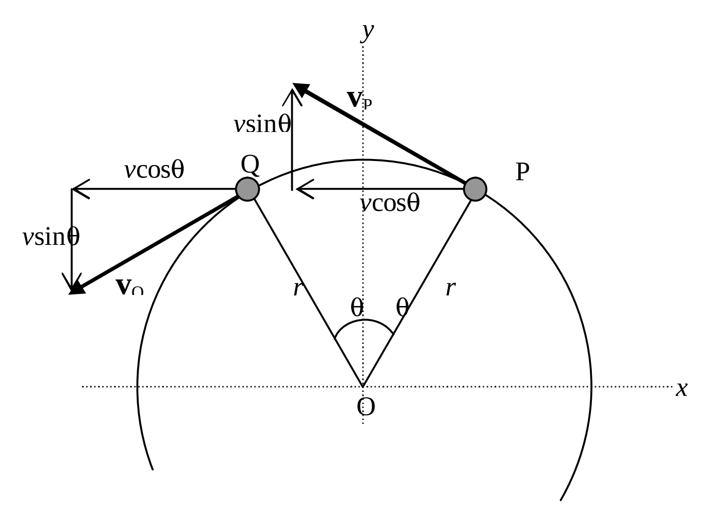


Figure 3.5 A particle moves in a circular motion of radius r. As $\theta \rightarrow 0$, $\Delta\mathbf{v}$, and consequently ***a*,** point toward the center.

3.3 UNIFORM CIRCULAR MOTION

When an object moves on a circular path with constant speed v, the object is said to be in a uniform circular motion. As the velocity is always tangent to the curve of the motion, the velocity, in a uniform circular motion, changes continuously in direction. As a result of this change in the velocity's direction, and although the speed is constant, the velocity as a vector quantity is changeable resulting in acceleration directed toward the center (**centripetal acceleration**). and is given as

$$a = \frac{v^2}{r} \qquad (3.16)$$

where r is the radius of the circular path.

To prove Equation (3.16) consider Figure 3.5. A particle moves in a uniform circular motion with constant speed v around a circle of radius r. The point P is the position of the particle at time t and the point Q, which is symmetric with P about the y-axis, is the position of the particle at $t+\Delta t$. The velocity at P is $\mathbf{v}_P$ (tangent to the curve at P) and the velocity at Q is $\mathbf{v}_Q$ (tangent to the curve at Q).
Now the average acceleration of the particle as it moves from P to Q is

$$\bar{a}_x = \frac{v\cos q - v\cos q}{\Delta t} = 0$$

and

$$\bar{a}_y = \frac{-v\sin q - v\sin q}{\Delta t} = \frac{-2v\sin q}{(2rq/v)}$$

In the last equation we have substituted for $\Delta t = \dfrac{2qr}{v}$, which is the length of the arc from P to Q divided by the speed v. As $\Delta t \rightarrow 0$, the angle $\theta \rightarrow 0$ and the ratio $(\sin q)/q \rightarrow 1$. The instantaneous acceleration at the top of the path is then

$$a = \lim_{\Delta t \to o} \frac{\Delta v}{\Delta t} = -\frac{v^2}{r}$$

Except the minus sign, this equation is exactly equation 3.16. The minus sign tells that the acceleration points toward the center.

Example 3.5 The moon revolves about the earth in an orbit (assuming it is a circular) of radius 3.85 x 10^5 km and makes one revolution in 27.3 days. Find the acceleration of the moon toward the earth.

Solution
The time for one revolution, called the period is

$$T = 27.3 \times 24 \times 3600 = 2.36 \times 10^6 \text{ s}.$$

The speed of the moon is therefore

$$v = \frac{2pr}{T} = 1026 \text{ m/s}.$$

The centripetal acceleration is now

$$a = \frac{v^2}{r} = \frac{(1020)^2}{3.85x10^8} = 2.73 \times 10^{-3} \text{m/s}^2 .$$

3.4 RELATIVE MOTION

In the previous chapter we discussed the relative motion in one-dimension. In this section we consider the same topic but this time in the two-dimensional case.

Consider that two persons want to observe the motion of a particle P. The first person, Ahmad in frame E, is stationary (with respect to the earth), while the second person, Mustafa in frame M, is moving with constant velocity $\mathbf{v}_{ME}$ relative to the earth. If the two reference frames coincide at $t=0$, then after a time t, the displacement of M relative to E will be $\mathbf{r}_{ME} = \mathbf{v}_{ME} t$. If the displacement of the particle as measured by E is $\mathbf{r}_{PE}$, and the displacement of the particle as measured by M is $\mathbf{r}_{PM}$, then as it is clear from Figure 3.6

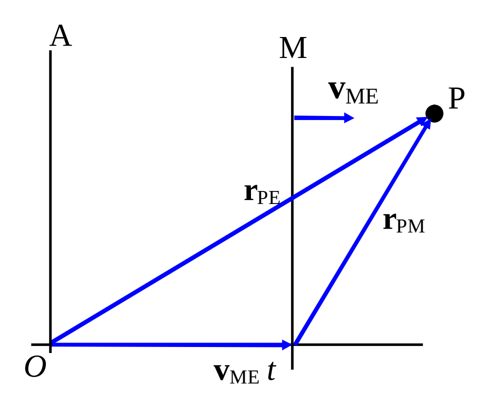


Figure 3.6 Ahmad (frame E) and Mustafa (frame M) observing a body (*P*). Ahmed is stationary while Mustafa is moving with a constant velocity $\mathbf{v}_{\mathrm{ME}}$ relative to the earth.

$$\mathbf{r}_{\mathrm{PE}} = \mathbf{r}_{\mathrm{PM}} + \mathbf{v}_{\mathrm{ME}} t \tag{3.17}$$

If we differentiate Equation (3.17) with respect to time, we get

$$\boxed{\mathbf{v}_{\mathrm{PE}} = \mathbf{v}_{\mathrm{PM}} + \mathbf{v}_{\mathrm{ME}}} \tag{3.18}$$

where $\mathbf{v}_{\mathrm{PA}}$ is the velocity of the particle relative to the reference frame A and $\mathbf{v}_{\mathrm{PM}}$ is the velocity of the particle relative to M. Equations 3.17 and 3.18 are known as **Galilean transformation equations**. To find the acceleration we have to differentiate Equation (3.18) with respect to time getting

$$\mathbf{a}_{\mathrm{PA}} = \mathbf{a}_{\mathrm{PM}}, \tag{3.19}$$

with $\mathbf{a}_{\mathrm{PA}}$ and $\mathbf{a}_{\mathrm{PM}}$ are the acceleration of the particle relative to A and M, respectively.

Example 3.6 A man, by a boat, wants to cross a river, 1500 m in wide, and flows due north with a speed of 1.5 m/s. The boat is rowed with a speed of 5 m/s due east relative to the water

a) What is the velocity of the boat relative to the ground?

b) Find the time needed for the boat to cross the river.

c) Determine the point the boat will reach on the opposite bank of the river.

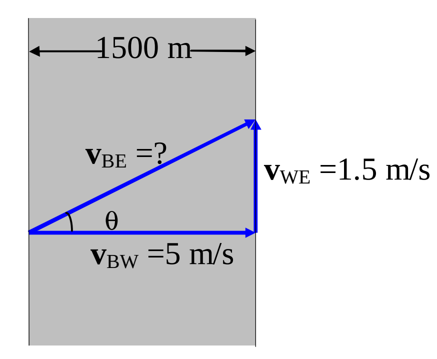


Figure 3.7 Example 3.6

Solution Let $\mathbf{v}_{WE}$, $\mathbf{v}_{BE}$, and $\mathbf{v}_{BW}$ be the velocity of the water relative to the earth, the velocity of the boat relative to the earth, and the velocity of the boat relative to the water, respectively.

a) As $\mathbf{v}_{BE} = \mathbf{v}_{BW} + \mathbf{v}_{WE}$ we obtain

$$\mathbf{v}_{BE} = 5\mathbf{i} + 1.5\mathbf{j} \text{ m/s}.$$

b) The wide of the river can be considered as the horizontal displacement of the boat., i.e. *x=1500* m, so

$$t = \frac{x}{(v_{BE})_x} = \frac{1500}{5} = 300\text{s}.$$

c) $$y = (v_{BE})_y t = 1.5(300) = 450\text{m},$$

i.e. the point is 450 m north of its starting point.

Example 3.7 In the previous example, if the boat have to reach a point on the opposite bank directly east from the starting point,
a) in what direction should the boat be headed.
b) what will be the velocity of the boat relative to the ground?
c) how much time is required to cross the river?

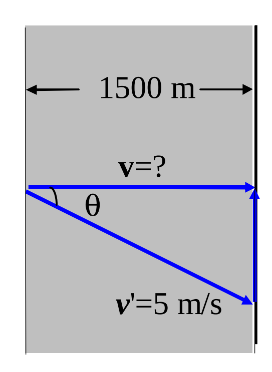


Figure 3.8 Example 3.7.

Solution a) For the boat to reach a point directly east from the starting point, the vertical displacement should be zero, i.e. $y = 0$ and this implies that $\mathbf{v}_{\mathrm{BE}} = v\mathbf{i}$. Now from Equation (3.18) we have

$$\mathbf{v}_{\mathrm{BE}} = \mathbf{v}_{\mathrm{BW}} + \mathbf{v}_{\mathrm{WE}},$$

$$v\mathbf{i} = \mathbf{v}_{BW} + 1.5\mathbf{j}$$

Therefore we obtain

$$\mathbf{v}_{BW} = v\mathbf{i} - 1.5\mathbf{j} \; .$$

As we have

$$v_{BW} = \sqrt{v^2 + (-1.5)^2} = 5 \text{ m/s},$$

so we obtain

$$v = \sqrt{25 - 2.25} = 4.77 \text{m/s}$$

And

$$\mathbf{v}_{BW} = 4.77\mathbf{i} - 1.5\mathbf{j}$$

Now

$$\tan q = \frac{1.5}{4.77} \Rightarrow q = 17.46^{o}$$

b) It is clear now that $\mathbf{v}_{BE} = v\mathbf{i} = 4.77\mathbf{i}$ m/s

c) To find the time of crossing we have

$$t = \frac{x}{(v_{BE})_x} = \frac{1500}{4.77} = 314.5\text{s}$$

PROBLEMS

3.1 The coordinates of a particle moving in the *x*-*y* plane vary with time as $\mathbf{r} = (2-4t)\mathbf{i} + (3t^2)\mathbf{j}$ with *r* in meters and *t* in seconds. Find
a) the velocity of the particle at *t*=3 s,
b) the acceleration of the particle at *t*=3 s,.

3.2 A car travels east a distance of 20 km, then travels north a distance of 30 km , and finally travels northwest a distance of 10 km. If the total trip lasts a period of 50 min., Find
a) the average speed of the car during the trip,
b) the average velocity of the car during the same period.

3.3 A particle starts from rest at the origin at *t*=0 and moves in the *x*-*y* plane with a constant acceleration given as $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}\,\mathrm{m/s^2}$. At *t*=2 s determine,
a) the position of the particle,
b) the velocity of the particle,
c) the speed of the particle.

3.4 A particle moves in the *x*-*y* plane with a velocity given by $\mathbf{v} = (2-3t)\mathbf{i} + 5t\,\mathbf{j}\ \mathrm{m/s}$. If the particle starts from the origin at *t*=0,
a) calculate the position of the particle at *t*=8 s,
b) calculate the acceleration of the particle.

3.5 A stone is thrown with an initial speed of 25 m/s at an angle of 37^o with the horizontal. Find
a) the range of the stone,
b) the maximum height reached by the stone.

3.6 A particle is launched with an initial speed of 40 m/s at an angle of 60^o above the horizontal. Calculate the magnitude and direction of its velocity after 1.5 s and 4.0 s.

3.7 A ball is thrown from a height with a speed of 16 m/s at an angle of 30^o below the horizontal.

a) Find the horizontal and the vertical displacements of the ball 2.0 s later.

b) Find the velocity of the ball at that instant.

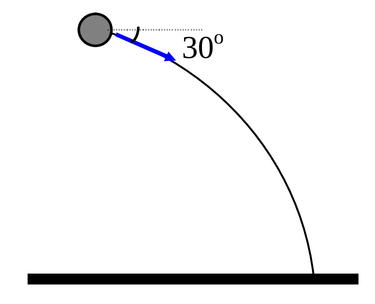


Figure 3.9 Problem 3.7

3.8 A projectile is projected with a speed of 70 m/s toward a building. The ball enters, horizontally, a window in the building that is 35 m above the point of projection.

a) What is the angle of projection?

b) What is the horizontal distance between the building and the point of projection?

c) With what speed the projectile enters the window.

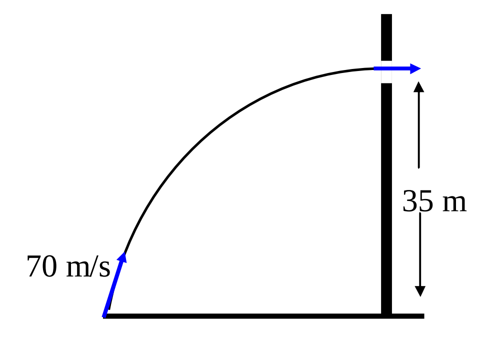


Figure 3.10 Problem 3.8

3.9 A projectile is thrown horizontally with a speed of 12 m/s from an edge of 60 m high.

a) Find the time of flight of the projectile.

b) Find the speed of the projectile just before hitting the ground.

3.10 A soccer player want to kick the ball from a point 30 m from the goal. The ball hits the crossbar, which is 3 m high. If the ball is kicked at an angle of 37^o,

a) Find the speed with which the ball was kicked.

b) Does the ball hit the crossbar while still rising or while falling?

3.11 A stone is projected toward a cliff of height 60 m with an initial speed of 45 m/s at an angle of 60^o above the horizontal. Calculate

a) the maximum height reached by the stone,

b) the velocity of the stone just before impact.

3.12 A ball is thrown horizontally from a height of 24 m and hits the ground with a speed that is three times its initial speed. What was the initial speed?

3.13 A plane traveling horizontally with a speed of 250 km/hr when a package is released from it. The horizontal distance between the point of release and the point where the package lands is 1.2 km.

a) How long was the package in the air?

b) How high was the plane when the package was released?

3.14 Ahmad throws a ball with a speed of 15 m/s at an angle of 30^o above the horizontal. His friend Omer, first standing beside him starts running 1 s after the ball is thrown. What average speed Omar must have in order to catch the ball just before it hits the ground?

3.15 A boy throws a stone at an angle of 53^o above the horizontal. The stone hits a building at a point 12 m above the point of throw. the building is 35 m away from the boy.

a) Find the speed with which the stone is thrown,

b) Find the velocity of the stone just before it hits the building.

3.16 A projectile is projected from the bottom of an incline with an initial velocity v_o at an angle q above the surface of the incline, as shown in Figure 3.11. If the incline is inclined at an angle f above the horizontal, show that the distance, measured along the incline, from the projection point to the point where the projectile hits the incline is

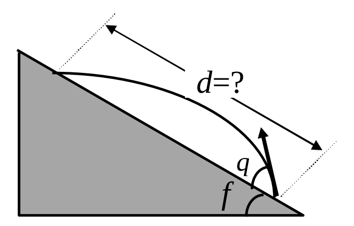


Figure 3.11 Problem 3.16

$$\frac{2v_o^2 \cos(q+f)\sin q}{g\cos^2 f}$$

3.17 A particle moves with constant speed of 6 m/s in a circular track of radius 1.5 m. Find the acceleration of the particle.

3.18 A particle moves with constant speed in a circular path makes six revolutions per second. If the radius of the path is 0.6 m, find

a) the speed of the particle,

b) the acceleration of the particle.

3.19 In one model of the hydrogen atom, the electron rotates about the proton in a circle of radius 0.53 A^o with a speed of 2.18×10^6 m/s. What is the acceleration of the electron?

3.20 The pilot of an airplane want to fly due north. A wind is blowing toward east at 100 km/hr. If the speed of the plane in still air is 300 km/hr,

a) In what direction should the pilot head?
b) What is the speed of the plane relative to the ground?

3.21 A bus travels with a speed of 100 km/hr on a straight highway is chased by a police car traveling at 105 km/hr. What is the velocity of the bus relative to the police car?

3.22 A man rows a boat across a river with a velocity, in still water, of 2 m/s due west. After 10 minutes he finds himself at a point that is 800 m west and 300 m north of his starting point. Calculate the water velocity, in magnitude and direction.

3.23 A car travels due east with a speed of 40 km/hr in a rainy day. The rain is falling vertically relative to the ground. A man in the car observes that the traces of the rain on the side windows of the car makes an angle of 53^o with the vertical.
a) Find the velocity of the rain relative to the car.
b) Find the speed of the rain relative to the ground.

3.24 A boy is riding a bicycle traveling with constant speed of 25 km/hr.
He wants to hit a target in front of him that is 12 m above the height of his hands by throwing a stone with a speed of 20 m/s relative to himself. The stone hits the target horizontally.
a) What is the projection velocity of the stone relative to the ground?
b) At what horizontal distance in front of the target must he release the stone?

CHAPTER 4

NEWTON'S LAWS OF MOTION

Up to now we have described the motion of particles using quantities like displacement, velocity and acceleration. These quantities are known as the **kinematics quantities**. In this chapter we will study what cause these quantities, i.e. the agent that causes the motion. To do so we have to introduce the concepts of force and mass and discuss the three laws of motion stated by Sir Isaac Newton (1686). These three laws, hereafter will be named as Newton's laws, are based entirely on experimental observation and can not be derived from more fundamental concepts. This branch of mechanics, that study the cause of the motion, is called **dynamics**.

4.1 FORCE

When a body affects a second body, we say that the first body exerts a force on the second. This means that only force can cause the body to change its state of uniform motion or rest. If you push a heavy box trying to move it, you may not succeed and the box is still at rest. The net force acting on the box here is zero since in addition to the pushing force there is another force resulting from the contact between the box and the floor (force of friction). The forces that result from the physical contact between the objects are called **contact forces** and the forces that do not require physical contact to affect are called **field forces**. The gravitational and the electromagnetic forces are examples of field forces.

The direction of the motion will be different whether you push or pull a body. This means that a direction for the force must be specified to study the motion of the body, i.e. ***the force is a vector quantity.*** The unit of the force in the SI unit system is Newton, abbreviated N. In the cgs unit system its unit is dyne with 1dyne=10^{-5} N, and in the British unit system, the force's unit is pound (lb).

4.2 NEWTON'S FIRST LAW

The law states that *an object continues in its state of rest or uniform motion until it is forced to change that state by an external force.* The frame in which Newton's first law is valid is called **inertial frame**, i.e., an observer in an inertial frame have to find the acceleration of an object, that moves with constant velocity in another frame, is also zero. From section 3.4 we can say that all accelerated frames are not inertial frames. The ground can be considered as an approximate inertial frame regardless of its rotation about the sun and about its own axis.

4.3 MASS AND INERTIA

Inertia is the property of matter that resists the change of its state and mass is the measure of this inertia. Suppose, for instance, that you have two barrels of equal sizes, one empty and the other filled of oil. If you want to roll the two barrels
along a rough, horizontal surface, the filled barrel would certainly, take more effort to get it moved. Similarly, if the two barrels are in motion, you would require more effort to bring the filled barrel to rest. Therefore we say that the filled barrel has more inertia (larger mass) than the empty barrel. The mass is a scalar quantity with a unit of kilogram (kg), in the SI unit system and gram (g) in the cgs unit system.

Remark There is a difference between mass and weight. While the mass of a body is the measure of inertia of that body (the same everywhere), its weight is the force exerted by gravity on it (depends on the body's position).

4.4 NEWTON'S SECOND LAW

This law states that *the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass, i.e.,*

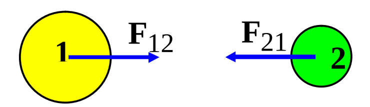


Figure 4.1 Newton's third law.

$$\sum \mathbf{F} = m\mathbf{a}, \tag{4.1}$$

where $\sum \mathbf{F}$ represents the net (resultant) force acting on the mass. Eq. 4.1 can be written in components form as

$$\sum F_x = ma_x, \tag{4.2a}$$
$$\sum F_y = ma_y, \tag{4.2b}$$
$$\sum F_z = ma_z. \tag{4.2c}$$

4.5 NEWTON'S THIRD LAW

The last law of Newton states that f*or every action there is an equal, but opposite reaction*, i.e., if two bodies interact, the force exerted by body number 1 on body number 2 ($\mathbf{F}_{21}$) is equal and opposite to the force exerted on body number 1 by body number 2 $\mathbf{F}_{12}$ (see Figure 4.1), namely

$$\mathbf{F}_{21} = -\mathbf{F}_{12} \tag{4.3}$$

Remark: Action and reaction act on different bodies.

Free-Body Diagram: It is a diagram showing all the forces acting on the body and do not include the forces the body exert on other bodies.

Strategy for solving problems using Newton's laws:

(i) Chose a suitable coordinate system with the positive direction is the direction of the acceleration, if it is known.
(ii) Draw a free-body diagram of each body of the system separately
(iii) Resolve each force into its components according to the chosen coordinates.
(iv) Identify the known and the unknown quantities.
(v) Now you can apply Newton's second law for one body or more of the system according to the unknown quantities.

Example 4.1 A boy want to drag a box, that has a mass of 3 kg, along a horizontal smooth surface. He pulls the box horizontally with a force of 2 N. Find the acceleration of the box.

Solution The free-body diagram of the system is shown in Figure 4.2, with the appropriate coordinate system. Now, from Newton's second law we have

$$a = \frac{F}{m} = \frac{2}{3} = 0.67 \text{ m/s}^2.$$

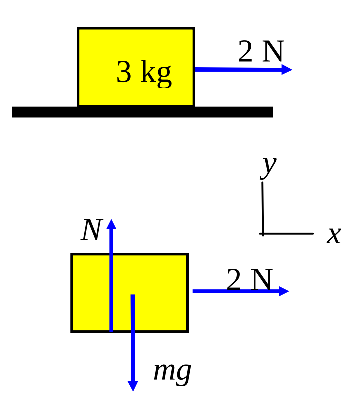


Figure 4.2 Example 4.1 with the free-body diagram of the box.

Note that $N = mg$, (why?). Here N is called the normal force, which is a force exerted on the box by the surface.

Example 4.2 Two masses m_1 and m_2 ($m_1 > m_2$) are suspended vertically by a light string that passes over a light, frictionless pulley as in Figure 4.3a (Atwood's machine). Find the acceleration of the masses and the tension in the string.

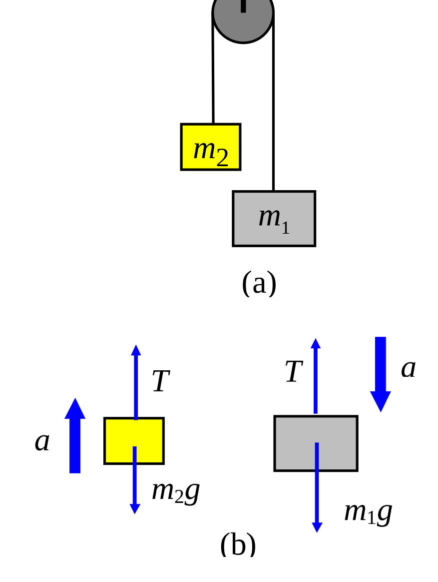


Figure 4.3 Example 4.2. (a) The Atwood's machine. (b) the free-body diagram of the two masses.

Solution The free-body diagram of the system is shown in Figure 4.3 (b), with the positive sense is taken downward. Applying Equation (4.1) for m_1, yield

$$m_1 g - T = m_1 a, \qquad (1)$$

and for m_2, we obtain

$$m_2 g - T = -m_2 a. \qquad (2)$$

Solving Equation (1) and Equation (2) for a and T, we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g,$$

and

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g.$$

Example 4.3 Two blocks are in contact on a smooth horizontal table. A constant force F is applied to one block as in Figure 4.4.

a) Find the acceleration of the system.

b) Find the contact force between the two blocks

Solution Applying Newton's second law for mass m_1 :

$$F - F_c = m_1 a \,. \qquad (1)$$

For mass m_2:

$$F_c = m_2 a \,. \qquad (2)$$

a) Adding Equations (1) and (2), you get

$$F = (m_1 + m_2)a \,,$$

or $$a = \frac{F}{m_1 + m_2}$$

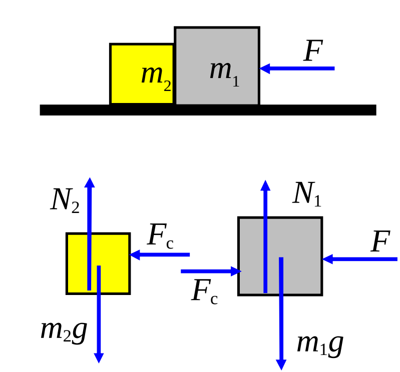


Figure 4.4 Example 4.3. with the free-body diagram of the two masses.

You can find the acceleration of the system by applying Newton's second law to the whole system.

b) Substituting for *a* in Equation 2, you obtain

$$F_c = \frac{m_2 F}{m_1 + m_2}$$

Example 4.4 A man of mass 80 kg stands on a platform scale in an elevator, as in Figure 4.5. Find the scale reading when the elevator
a) moves with constant velocity,
b) ascends with an acceleration of 3 m/s^2,
c) descends with an acceleration of 3 m/s^2

Solution Noting that the scale reads its reaction force, and applying Newton's second law in the three cases, you get

a) $$\sum F = N - mg = 0,$$
or
$$N = mg = 784\text{N}.$$

So the scale will read the actual weight of the man.

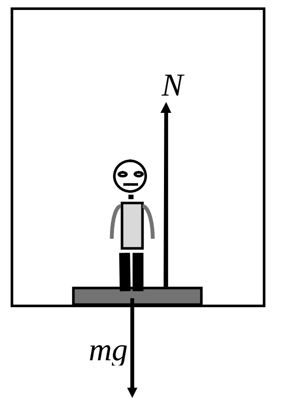


Figure 4.5 Example 4.4.

b) $$\sum F = N - mg = ma,$$

or

$$N = m(g + a) = 1024\text{N}$$

c) $$\sum F = N - mg = -ma,$$
or

$$N = m(g - a) = 544\text{N}$$

4.6 FRICTIONAL FORCES

One of the most commonly encountered types of force are the frictional forces: The forces that two surfaces in contact exert on each other to oppose the sliding of one surface over the other. This kind of force results mostly from the interaction between the atoms and the molecules on the surfaces.

Consider that you want to push your textbook along a horizontal table (Figure 4.6). Regarding the frictional force, three cases have to be considered:

Case 1: Before starting pushing, the book is initially at rest, and this means that its acceleration is zero. As the force of friction is the only force acting horizontally on the book, this force in this case is equal to zero.

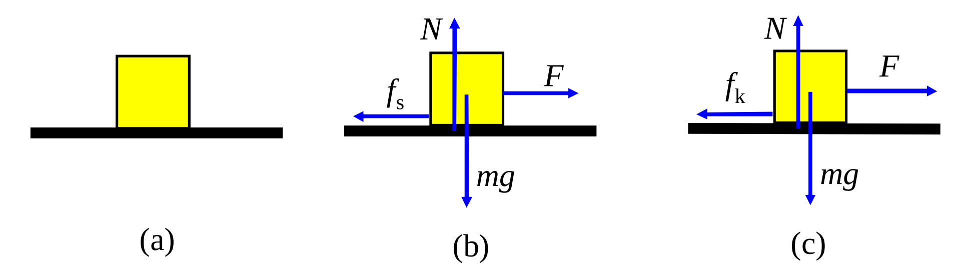


Figure 4.6 (a) Case 1: f_s=0. (b) Case 2: $f_s = F \le \mu_s N$. (c) Case 3: $f_k = \mu_k N$.

Case 2: If you push the book gently, the book will not move and remains at rest. Here, there are two forces acting horizontally on the book: the pushing force F and the frictional force. For the acceleration to be zero, these two forces must be equal in magnitude and opposite in direction. This force is called the force of **static friction** and will be denoted hereafter by f_s.

Case 3. Now, slowly increasing the pushing force, the book will begin to move when the pushing force reaches a critical value f_s(*max.*). Once in motion, the frictional force is less than f_s(*max.*), and is called the force of **kinetic-friction** , denoted by f_k.

It is found, experimentally, that the frictional forces f_s and f_k, between two surfaces, are proportional to the normal force N pressing the two surfaces together. i.e.,

$$f_s \le m_s N , \qquad (4.4)$$

and

$$f_k = m_k N . \qquad (4.5)$$

Where the dimensionless constants μ_s and μ_k are, respectively, the coefficient of static friction and the coefficient of kinetic friction.

Remarks: 1- The frictional force is always parallel to the surfaces in contact.
2- The force of static-friction is always opposite to the applied force
3- The force of kinetic friction is always opposite to the direction of motion.
4- The frictional force, together with the normal force constitute the two perpendicular components of the reaction force exerted by one of the contact bodies on the other.

Example 4.5 A block of mass *m* slides down a rough, inclined plane with the angle of inclination is θ as shown in Figure 4.7. The coefficient of kinetic friction between the block and the plane is μ.
a) Find the acceleration of the block.
b) If the block starts from rest at the top of the plane, find its velocity after it slides a distance *d* along the plane.

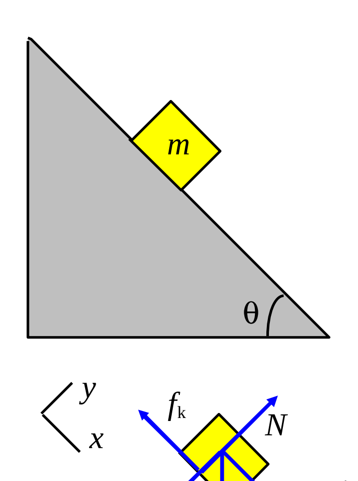


Figure 4.7 Example 4.5 with the free-body diagram of the block.

Solution: The free-body diagram of the block is shown in Figure 4.7. Note that the *x*-axis is chosen along the plane.
a) Now applying Newton's second law in the *y*-axis, we get

$$\sum F_y = N - mg\cos\theta = 0,$$

and in the *x*-axis, we obtain

$$\sum F_x = mg\sin q - f_k = ma .$$

Substituting for $f_k = m_k N = m_k mg\cos q$, we have

$$a = g(\sin q - m\cos q)$$

b) Since the acceleration is constant, we use Equation (2.11) to get

$$v^2 = v_o^2 + 2ax = 0 + 2gd(\sin q - m\cos q)$$

or

$$v = \sqrt{2gd(\sin\theta - \mu\cos\theta)}$$

Example 4.6 A worker drags a crate along a rough, horizontal surface by pulling on a rope tied to the crate. The worker exerts a force of 300 N on the rope that is inclined 37^o to the horizontal as shown in Figure 4.8. If the mass of the crate is 60 kg, and the coefficient of kinetic friction between the crate and the surface is 0.3, find the acceleration of the crate.

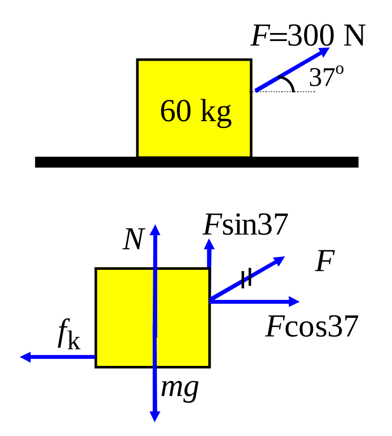


Figure 4.8 Example 4.6 with the free-body diagram of the system.

Solution First we construct the free-body diagram of the system as shown in Figure 4.8. After resolving the applied forces into its components according to the chosen axes, we apply Newton's second law to get, in the *y*-axis

$$\sum F_y = N + 300\sin 37 - mg = 0$$

from which we find

$$N = 588 - 181 = 407\,\text{N}\,.$$

Now applying Newton's second law in the x-axis, we obtain

$$\sum F_{\text{x}} = 300\cos 37 - f_{\text{k}} = ma\,.$$

But

$$f_{\text{k}} = m_{\text{k}} N = 0.3(407) = 122.1\,\text{N}\,,$$

so

$$a = \frac{117.5}{60} = 1.96\ \text{m/s}^2$$

Example 4.7 A 2-kg block is placed on top of a 5-kg block as shown in Figure 4.9. A horizontal force of $F = 40\,\text{N}$ is applied to the 5-kg block. If the coefficient of kinetic friction between the 5-kg block and the surface is 0.2, and assuming that the 2-kg block is in the verge of slipping,
a) what is the acceleration of the system?
b) What is the coefficient of static-friction between the two blocks?.

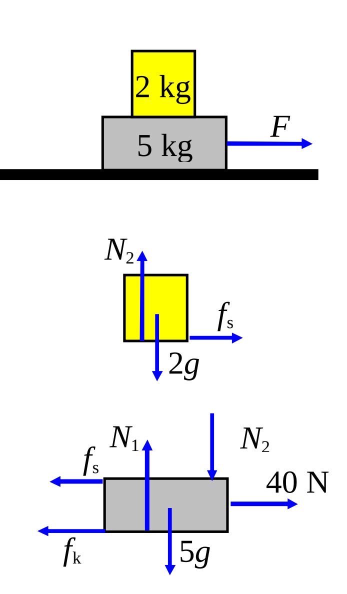


Figure 4.9 Example 4.7 with the free-body diagram of the two blocks. Note that the static frictional force is the force that accelerate the upper block.

Solution Study carefully the free-body diagram of the two blocks (Figure 4.9). The normal force acting on the 5-kg block by the surface is denoted by N_1, while N_2 stands for the normal force acting on the upper block by the lower one.

a) Applying Newton's second law to the 2-kg block, we have in the *x*-axis

$$f_s = 2a\,, \qquad 1$$

and in the *y*-axis

$$N_2 - 2g = 0$$

or

$$N_2 = 2g = 19.6\text{N} \qquad 2$$

Similarly for the 5-kg block, we have

$$\sum F_x = F - f_k - f_s = 5a\,, \quad 3$$

and

$$\sum F_y = N_1 - 5g - N_2 = 0$$

or

$$N_1 = 7g = 68.6\text{N}\,, \qquad 4$$

where the value of N_2 is taken from Equation 2. Note that, the two blocks have the same acceleration because the 2-kg block does not slip. Now, Adding Equation 1 and Equation 3 , we obtain

$$F - m_k N_1 = 7a\,,$$

where we have substituted for $f_k = \mu_k N_1$. Now substituting for N_1 from Equation 4, we have

$$a = \frac{40 - 0.2 \times 68.6}{7} = 3.75 \,\mathrm{m/s^2}\,.$$

b) Substituting for *a* in Equation 1, we get

$$m_s = \frac{2a}{N_2} = \frac{2 \times 3.75}{19.6} = 0.38\,.$$

PROBLEMS

4.1 Two forces act on a 2-kg mass. The first is $\mathbf{F}_1 = (4\mathbf{i} + 2\mathbf{j})\,\text{N}$, and the second is $\mathbf{F}_2 = (6\mathbf{i} - 4\mathbf{j})\,\text{N}$.
a) What is the net force acting on the mass?
b) What is the acceleration of the mass?

4.2 What is the net force acting on a 1800 kg automobile accelerating at 8 m/s^2.

4.3 A boy pushes a 12-kg box along a horizontal surface with a force of 50 N that makes an angle of 37^o with the horizontal.
a) Calculate the acceleration of the box.
b) What is the normal force acting on the box by the surface?

4.4 A block of weight 60 N rests on a smooth, horizontal table. A rope is tied to the block while the free end of the rope is held by a man such that the rope makes an angle of 30^o with the horizontal, as shown in Figure 4.10. What is the force exerted on the block by the table if the tension on the rope is,
a) 0 **b)** 40 N **c)** 130 N

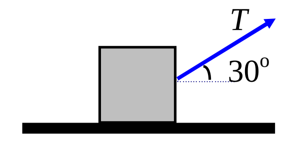


Figure 4.10 Problem 4.4.

4.5 Two blocks of masses $m_1 = 5\,\text{kg}$, and $m_2 = 8\,\text{kg}$ are suspended vertically by a light string that passes over a light, frictionless pulley as shown in Figure 4.11.
a) Calculate the acceleration of the system.

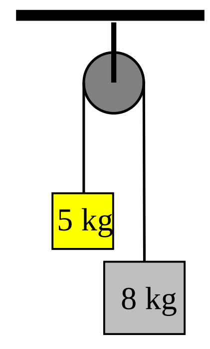


Figure 4.11 Problem 4.5.

b) Find the tension in the string.

4.6 A 800-kg car moving at 60 km/h is braked and brought to rest in a distance of 50 m. Calculate the fictional force during this distance.

4.7 A 12-kg block rests on a frictionless horizontal table. A second block of 2 kg tied to the first block by a light rope that passes over a light pulley as shown in Figure 4.12. A horizontal force F is applied to the 12-kg block.
a) For what value of F will the 2-kg block accelerate upward?
b) If F=30 N, calculate the acceleration of the masses.

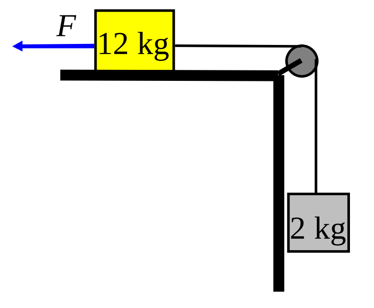


Figure 4.12 Problem 4.7.

4.8 An object is hung from a spring balance attached to the ceiling of an elevator. When the elevator is at rest the balance reads 50 N.
a) What does the balance read when the elevator is moving up with a constant acceleration of 8 m/s^2?
b) What is the reading of the balance when the elevator has an acceleration of 6 m/s^2 downward?

4.9 A man of mass 70 kg is falling through the air with his parachute open. He experiences a downward acceleration of 2.5 m/s^2. If the mass of the parachute is 4.0 kg, calculate
a) The force exerted on the parachute by the air.
b) The force exerted by the man on the parachute.

4.10 Two blocks, m_1 and m_2, connected by a rope, rest on a frictionless, horizontal surface. The blocks are pulled to the right by applying a

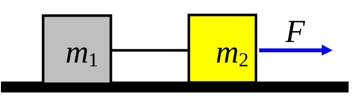


Figure 4.13 Problem 4.10.

force F to one of the masses as shown in Figure 4.13. Calculate

a) the acceleration of the system,

b) the tension in the rope.

4.11 In the previous problem (problem 4.10), if $m_1 = 2\,\text{kg}$, and $m_2 = 4\,\text{kg}$, $F = 80\,N$ and the horizontal surface is rough with $\mu_k = 0.3$, find the acceleration of the system and the tension in the rope.

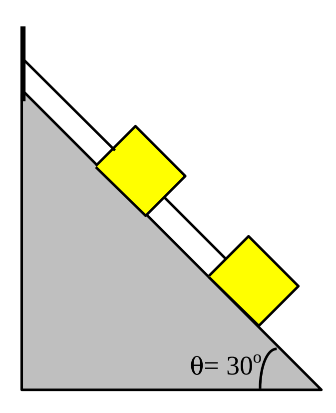


Figure 4.14 Problem 4.12.

4.12 Two blocks, each of mass 5 kg are held in equilibrium on a frictionless incline by two ropes as shown in Figure 4.14. Calculate the tension in

a) the rope connecting the two blocks,

b) the rope that connects the upper block to the wall.

4.13 Consider the system shown in Figure 4.15. The coefficient of kinetic friction between the 4-kg block and the table is 0.2.

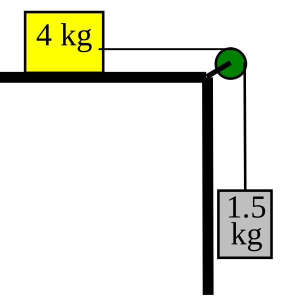


Figure 4.15 Problem 4.13.

a) Calculate the acceleration of the masses.
b) Calculate the tension in the string.

4.14 A block of mass $m_1 = 8\,\text{kg}$, rests on a frictionless inclined plane of angle 30^o, is connected, by a string over a light frictionless pulley, to a second block of mass $m_2 = 2\,\text{kg}$ hanging vertically (Figure 4.16).



a) What is the acceleration of the masses?



b) What is the tension in the string?

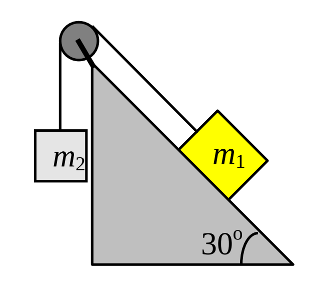


Figure 4.16 Problem 4.14.

4.15 A block of mass m=4 kg is at rest on an inclined, rough surface as shown in Figure 4.17. The angle of inclination is $\theta=30^o$ and the coefficients of frictions are $m_s = 0.7$ and $m_k = 0.5$. find the frictional force acting on the block.

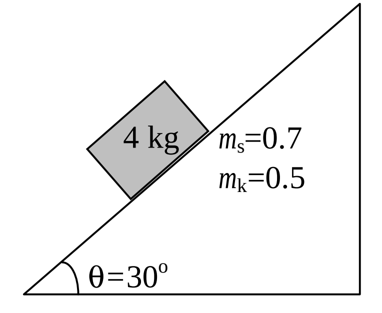


Figure 4.17 Problem 4.15

4.16 A horizontal force F of 40 N holds a block of 2 kg against a vertical wall as shown in Figure 4.18. If the coefficient of static friction between the block and the wall is $\mu_s = 0.6$, and the coefficient of kinetic friction is $\mu_k = 0.4$. What is the force exerted on the block by the wall?

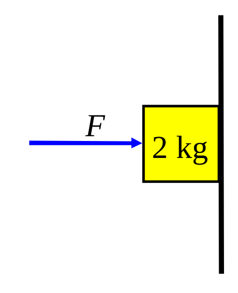


Figure 4.18 Problem 4.16.

4.17 Three different masses $m_1 = 1\text{kg}$, $m_2 = 3\text{kg}$, and $m_3 = 5\text{kg}$ are in contact with each other on a frictionless, horizontal surface. A horizontal force F= 45 N is applied to m_1 as shown in Figure 4.19.

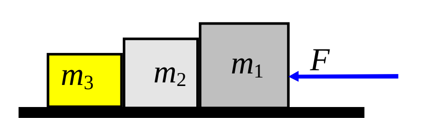


Figure 4.19 Problem 4.17.

a) Find the acceleration of the blocks.
b) Find the magnitude of the contact forces between the blocks.

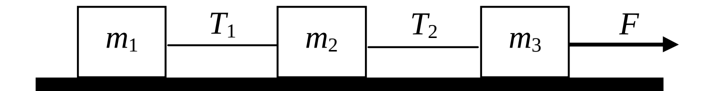


Figure 4.20 Problem 4.18.

4.18 Three blocks of masses $m_1 = 4\text{kg}$, $m_2 = 6\text{kg}$, and $m_3 = 8\text{kg}$ are connected by two ropes on horizontal smooth surface, as shown in Figure 4.20. A horizontal force F= 90 N is applied to m_3. Calculate
a) the acceleration of the system,
b) the tensions T_1 and T_2.

4.19 The two blocks in Figure 4.21 are connected by a string that passes over a frictionless, light pulley. The acceleration of the system is 1.5 m/s^2 to the right, and the surfaces are rough.
a) What is the tension in the string?
b) If the coefficient of kinetic friction is the same between the two blocks and the surfaces, find its value.

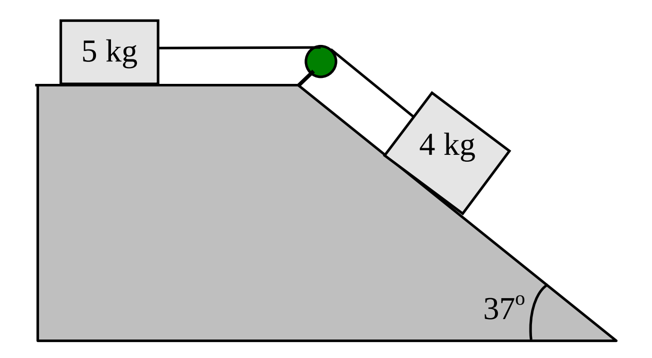


Figure 4.21 Problem 4.19.

4.20 A 8-kg block is placed in top of another block of 32 kg., which in turn rests on a horizontal, frictionless surface as shown in Figure 4.22. The coefficient of static friction between the blocks is $\mu_s = 0.6$, whereas the kinetic coefficient is $\mu_k = 0.4$. A horizontal force of 60 N is applied to the 8-kg block. Calculate the acceleration of

a) the 8-kg block, **b)** the 32-kg block.

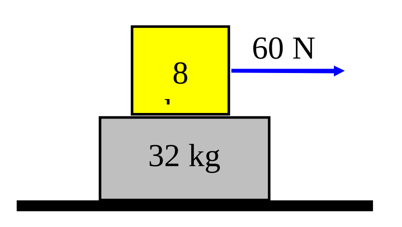


Figure 4.22 Problem 4.20.

4.21 A 6-kg mass is placed on top of a 12-kg block. A horizontal force of 100 N is applied to the 12-kg block, while the 6-kg mass is tied to the wall as shown in Figure 4.23. The coefficient of kinetic friction between all the surfaces is 0.3.

a) Determine the tension in the string.

b) Compute the acceleration of the 12-kg block.

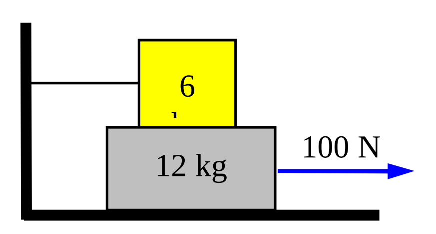


Figure 4.23 Problem 4.21.

4.22 A mass *m* slides down an inclined plane of angle 37° with constant velocity while a another identical mass rests on top of the first mass as shown in Figure 4.24. A cord attaches the upper mass to the top of the plane. If the coefficient of friction between all the surfaces is the same, find its value.

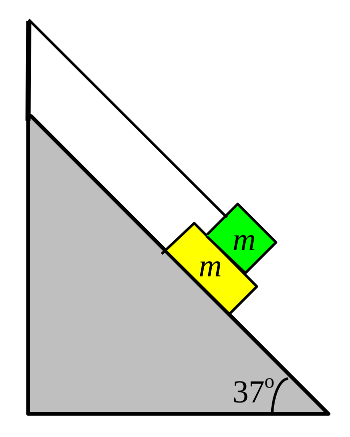


Figure 4.24 Problem 4.22.

4.23 A block of mass 8.0 kg is on a smooth, inclined plane of an angle of 53°. The block is pushed up the plane at constant speed by a horizontal force *F*, as shown in Figure 4.25.

a) What is the magnitude of *F*?

b) What is the force acting on the block by the plane?

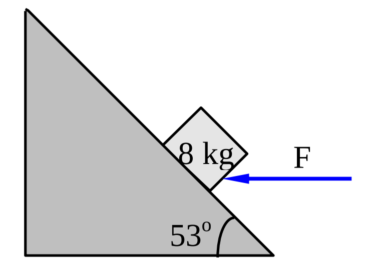


Figure 4.25 Problem 4.23.

4.24 A block of mass 2 kg is projected up an inclined plane with an initial speed of 12 m/s. The coefficients of friction between the block and the plane are $m_k = 0.3$ and $m_s = 0.6$, (Figure 4.26).

a) How far up the plane will the block go

b) After reaching its highest point, will the block stay there or slides back down?

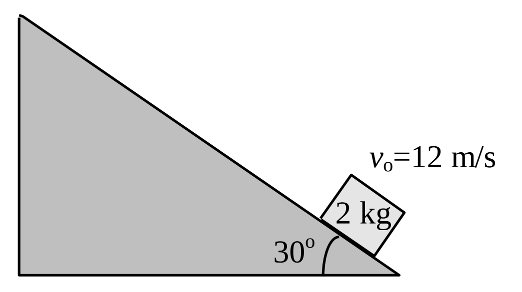


Figure 4.26 Problem 4.24.

4.25 In Figure 4.27, block B weighs 80 N and the coefficient of static friction between the block B and the table is 0.4. What is the maximum weight the block A can have such that the system be in equilibrium?

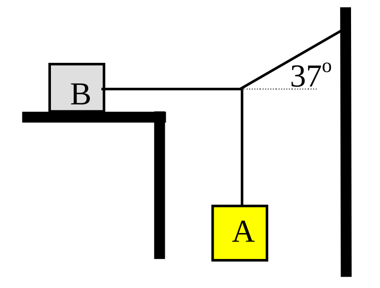


Figure 4.27 Problem 4.25.

4.26 What acceleration must the cart in Figure 4.28 have in order that the block not falls? The coefficient of static friction between the block and the cart is μ_s.

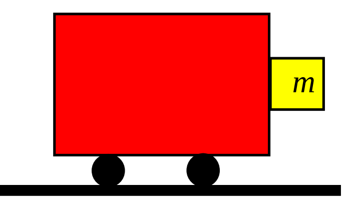
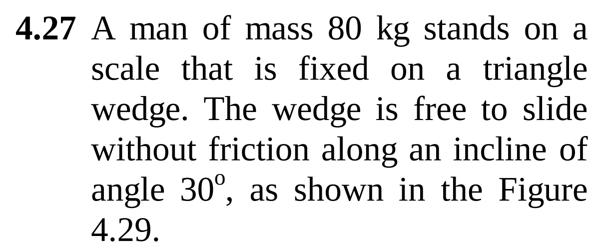


Figure 4.28 Problem 4.26.



4.27 A man of mass 80 kg stands on a scale that is fixed on a triangle wedge. The wedge is free to slide without friction along an incline of angle 30°, as shown in the Figure 4.29.

a) Draw a free-body diagram for the man.

b) Find the reading of the scale if the man is fixed relative to the wedge.

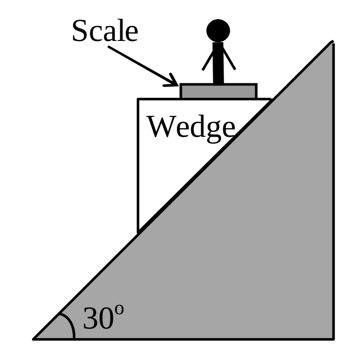


Figure 4.29 Problem 4.27.

CHAPTER 5

CIRCULAR MOTION AND GRAVITATION

In the previous chapter we discussed Newton's laws of motion and its application in simple dynamics problems. In this chapter we continue our study of dynamics and the applications of Newton's laws, specially on circular motion. Newton's law of gravitation is also addressed which is the central law in planet and satellite motion.

5.1 CENTRIPETAL FORCE

In section 3.3 we have showed that, if a particle moves with constant speed *v* in a circular path of radius *r*, it acquires a centripetal acceleration due to the change in the direction of the particle's velocity. The direction of the centripetal acceleration is always toward the center of the path and its magnitude is given by

$$a_r = \frac{v^2}{r}. \tag{5.1}$$

The subscript *r* referring to the radial component of the acceleration. According to Newton's second law this acceleration should be a result of an applied force acting on the particle toward the center of the path, and should have a magnitude of

$$F_r = ma_r = m\frac{v^2}{r}. \tag{5.2}$$

Because of its direction such a force is called the **centripetal force**. Both the centripetal acceleration and the centripetal force are vector quantities whose magnitudes are constant but whose directions are always changing so as to point toward the center of the circular path.

It should be noted that any force in nature can be treated as a centripetal force if it acts on a particle in a direction toward the center of the circular path followed by the particle. The frictional

force is the centripetal force when a car rounding a curve, and the tension is the centripetal force when you whirl a ball, tied to a string, in a horizontal circle.

Remark: The word centripetal referring to a specific direction of the force and not to a new kind of forces. It is like horizontal or vertical.

Example 5.1 A flat (unbanked) curve on a highway has a radius of 100 m. If the coefficient of static-friction between the tires and the road is 0.2, what is the maximum speed with which the car will have in order to round the curve successfully?.

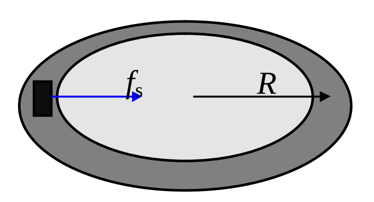


Figure 5.1 Example 5.1.

Solution Here there are three forces acting on the car: The weight and the normal force act perpendicular to the plane of motion, and the static frictional force which must be parallel to the road. Hence the centripetal force on the car is the force of static friction, so we have

$$f_s = m_s N = m\frac{v^2}{R},$$

but, since there is no motion in the vertical direction we can write

$$N = mg\,.$$

Solving the two equations for v we get

$$v = \sqrt{m_s g R} = 50.4\text{km/hr}.$$

Example 5.2 A circular curve of a road is designed for traffic moving at 60 km/hr without depending on the friction. If the radius of the curve is 80 m, what is the correct angle of banking of the road.

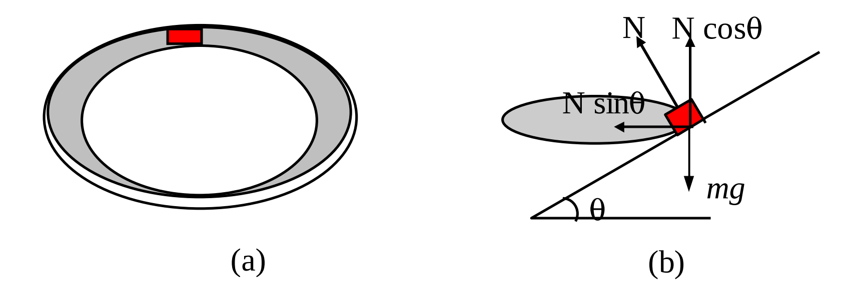


Figure 5.2 Example 5.2, with the free body diagram.

Solution In the banked roads, the normal force N should be resolved into two components: one toward the center of the curve (horizontal), and the other vertical as shown in Figure 5.2(b). The centripetal force will be then the component $N\sin q$, i.e.,

$$N\sin q = m\frac{v^2}{R},$$

and, since there is no motion in the vertical direction we have

$$N\cos q = mg .$$

Substituting for N from the second equation into the first equation, we have

$$mg \tan q = m\frac{v^2}{R}$$

or

$$q = \tan^{-1}\frac{v^2}{gR} = 19.5^o$$

Example 5.3 A ball of mass 1 kg is attached to one end of a string 1 m long and is whirled in a horizontal circle, as shown in Figure 5.3. Find the maximum speed the ball can attain without breaking the string. The breaking strength of the string is 500 N.

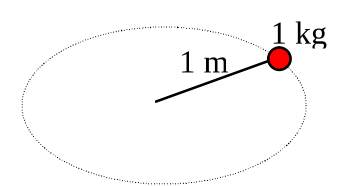


Figure 5.3 Example 5.3.

Solution The only two forces acting on the ball are the weight and the tension. Since the weight is normal to the plane of the circle, the centripetal force in this case is the tension, so we can write

$$T = m\frac{v^2}{R}.$$

To find the speed at the verge of breaking, we have to substitute for T by its breaking value, i.e.,

$$v_{\max} = \sqrt{\frac{T_{\max} R}{m}} = \sqrt{\frac{500 \times 1}{1}} = 22.4 \text{ m/s}$$

5.2 NONUNIFORM CIRCULAR MOTION

In the previous section we have considered the circular motion with constant speed (uniform circular motion). When the magnitude of the velocity is not constant but rather change with time we have the nonuniform circular motion. Now what will happen if the velocity changes both in magnitude and in direction. The change in the speed will add another contribution to the acceleration. Resolving the acceleration vector into two perpendicular components: radial component and tangential component, we can write

$$\mathbf{a} = a_{\mathrm{r}}\hat{\mathbf{r}} + a_q\hat{\mathrm{q}}\,, \tag{5.3}$$

where $\hat{\mathbf{r}}$ is a unit vector directed along the radius of the circular path, and q is another unit vector tangent to the path. the radial component, a_{r}, is the centripetal acceleration defined previously, and the tangential component, a_q, is the new contribution due to the change in the magnitude of the particle's velocity, so we will expect

$$a_q = \frac{d|\mathbf{v}|}{dt}\,. \tag{5.4}$$

Remark: In applying Newton's second law for the circular motion, the coordinate axes will be the radius-axis and the tangent-axis, so all the applied forces have to be resolved accordingly. The law now reads

$$F_{\mathrm{r}} = ma_{\mathrm{r}}\,, \text{ and } F_q = ma_q\,. \tag{5.5}$$

The positive senses of $\hat{\mathbf{r}}$ and q will be chosen toward the center, and counterclockwise respectively.

Example 5.4 A small body of mass m swings in a vertical circle at the end of a cord of length L as shown in Figure 5.4. If the speed of the body when the cord makes an angle θ with the vertical is v, find

a) the radial and the tangential components of the acceleration at this point,

b) the tension in the cord at the same point.

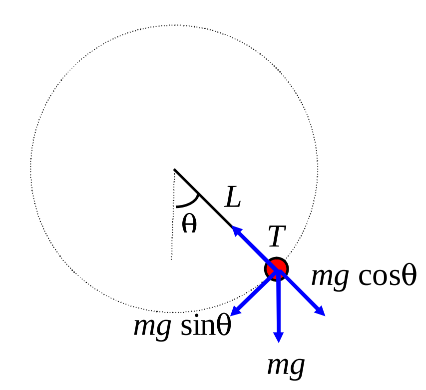


Figure 5.4 Example 5.4.

Solution The weight have to be resolved as shown in the free-body diagram of the system. As it is clear from the diagram, the radial component is

a) $$a_r = \frac{v^2}{R} = \frac{v^2}{L},$$

and the tangential component is

$$a_\theta = \frac{F_\theta}{m} = g\sin\theta\ .$$

b) Since $F_r = ma_r$, we have

$$T - mg\cos\theta = m\frac{v^2}{L}$$

or

$$T = m(g\cos\theta + \frac{v^2}{L})$$

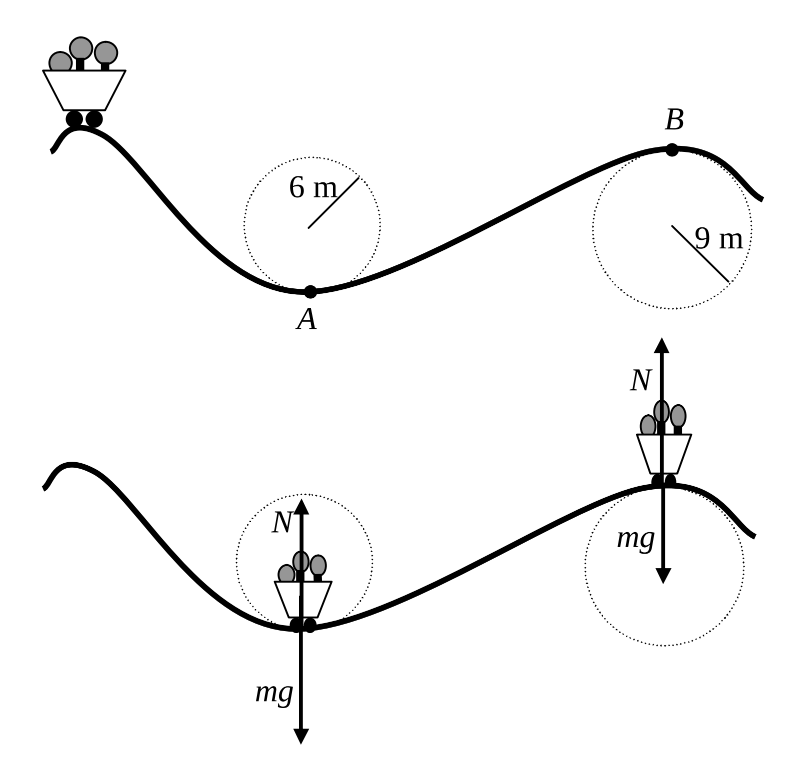


Figure 5.5 Example 5.5.

Example 5.5 A vehicle of mass 350 kg moves on a roller-coaster as shown in Figure 5.5.
a) If the speed of the vehicle at point *A* is 18 m/s, what is the normal force the track exerts on the vehicle?
b) What is the maximum speed for the vehicle to remain on track at point *B*?

Solution a) Examining the free-body diagram of the vehicle at point A we see that N is toward the center , while mg is away from the center. Applying the equation

$$F_r = ma_r = m\frac{v^2}{r}$$

We obtain

$$N - mg = m\frac{v^2}{r}, \text{ or}$$

$$N = m\left(\frac{v^2}{r} + g\right) = 2.23 \times 10^3 \text{ N}$$

b) For the vehicle to be on track, the normal force must have a positive value, that is, $N > 0$. Now from the free body diagram of the vehicle at point b we write

$$mg - N = m\frac{v^2}{r}, \text{ or}$$

$$N = m\left(g - \frac{v^2}{r}\right) > 0$$

This leads to

$$v < \sqrt{gr}$$

So we get $v_{\text{max}} = \sqrt{gr} = 9.39 \text{ m/s}$

5.3 NEWTON'S LAW OF GRAVITATION

The law states that *every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them*. Thus the gravitational force exerted on a particle of mass m_1 by a particle of mass m_2 is

$$\mathbf{F}_{12} = G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}\,, \qquad (5.6)$$

where r_{12} is the distance between the two particles and $\hat{\mathbf{r}}$ is a unit vector directed from m_1 to m_2. The universal constant G is called the gravitational constant with a value, in SI units of

$$G = 6.672\times10^{-11}\ \mathrm{N.m^2/kg^2}.$$

It can be shown that the force exerted by any homogeneous sphere is the same as if the entire mass of the sphere is concentrated at its center. Therefore, the force exerted by the earth on a small body of mass m, a distance r from its center, is

$$F = G\frac{M_e m}{r^2}\,,\ \mathrm{r}>R_e \qquad (5.7)$$

where M_e and R_e are the earth's mass and the earth's radius, respectively. This force is directed toward the center of the earth. Inside the earth, the force would decrease as approaching the center rather than increasing as $\frac{1}{r^2}$. At the center of the earth the gravitational force on the body would be zero, why?

For freely falling body the only force acting is the gravitational force of the earth and the acceleration produced is the

acceleration due to gravity, *g*. Now , from Newton's second law, and assuming the body to be at the surface of the earth, we have

$$\sum F = G\frac{M_e m}{R_e^2} = mg\ , \tag{5.8}$$

or

$$g = G\frac{M_e}{R_e^2}\ . \tag{5.9}$$

The mass of the earth can be calculated using Equation (5.9) as

$$M_e = \frac{R_e^2 g}{G} = 5.96\times 10^{24}\ \text{kg},$$

with R_e=6370 km

The force acting on a particle at a distance *h* above the earth's surface is, from Equation (5.6) and Equation (5.8)

$$F = G\frac{M_e m}{(R_e + h)^2} = mg'\ , \tag{5.10}$$

or

$$g' = G\frac{M_e}{(R_e + h)^2}\ . \tag{5.11}$$

Therefore, g' decrease with increasing altitude.

Example 5.6 Two bodies of mass 60 kg, and 80 kg are placed 2 m apart. Calculate the gravitational force exerted by one body on the other.

Solution From Equation (5.5) we have

$$F = G\frac{m_1 m_2}{r^2} = \left(6.67\times10^{-11}\right)\frac{(60)(80)}{(2)^2}$$
$$= 8\times10^{-8}\,\text{N}.$$

Example 5.7 Three bodies of mass 2 kg, 4 kg, and 6 kg are arranged as shown in Figure 5.6. Calculate the total force acting on the 2-kg mass by the other two masses.

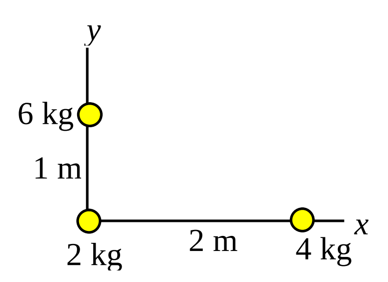


Figure 5.6 (Example 5.7).

Solution The force exerted on the 2-kg mass by the 4-kg mass is

$$\mathbf{F}_{24} = G\frac{m_2 m_4}{r_{24}^2}\mathbf{i} = \left(6.67\times10^{-11}\right)\frac{2\times4}{(2)^2}\mathbf{i}$$
$$= 1.33\times10^{-10}\,\mathbf{i}\,\text{N},$$

and the force exerted by the 6-kg is

$$\mathbf{F}_{26} = G\frac{m_2 m_6}{r_{26}^2}\mathbf{j} = \left(6.67\times10^{-11}\right)\frac{2\times6}{(1)^2}\mathbf{j}$$
$$= 8.0\times10^{-10}\,\mathbf{j}\text{N}.$$

Therefore, the total force acting on the 2-kg mass due to the 4-kg and the 6-kg masses is the vector sum of $\mathbf{F}_{24}$ and $\mathbf{F}_{26}$:

$$\mathbf{F}_2 = \mathbf{F}_{24} + \mathbf{F}_{26} = (1.33\mathbf{i} + 8.0\mathbf{j})\times10^{-10}\,\text{N}$$

Example 5.8 Calculate the magnitude of the acceleration due to gravity at an altitude of 100 km.

Solution From Equation (5.10) we have

$$g' = G\frac{M_e}{(R_e + h)^2}$$
$$= \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.37\times10^6 + 1\times10^5)^2}$$
$$= 9.5 \text{ m/s}^2.$$

This means that g' is decreased by a 3%.

5.4 SATELLITE MOTION

From section 3.2 we show that if one launch a projectile from the surface of the earth with a rather small velocity, the trajectory will be a parabola provided that the air resistance is neglected. By increasing the velocity of projection we can increase the size of the trajectory, and above a certain critical value of the velocity, the trajectory will miss the earth and the projectile has become an earth satellite. In this case the force acting on the projectile is no longer constant but varies inversely as $\frac{1}{r^2}$, with r is the radius of the satellite's orbit . It turns out that under such an attractive force, the projectile's path may be a circle, an ellipse, a parabola, or a hyperbola. The circular path will be considered for simplicity.

We have learned that a particle in a uniform circular motion has a centripetal acceleration given by $a_r = \frac{v^2}{r}$, with v is its speed

and r is the radius of the circular path. In satellite motion the gravitational force (Equation 5.7) is the force that provides such acceleration, that is

$$G\frac{M_{\mathrm{e}}m}{r^2} = m\frac{v^2}{r},$$

where m is the mass of the satellite and r is the radius of the satellite orbit. Solving for v we get

$$v = \sqrt{\frac{GM_{\mathrm{e}}}{\mathrm{r}}}\,. \tag{5.12}$$

The period of revolution is

$$t = \frac{2p\,r}{v}$$

By substituting for v from Equation 5.12, we get

$$t = 2p\sqrt{\frac{r^3}{GM_{\mathrm{e}}}} = \frac{2p\,r^{3/2}}{\sqrt{GM_{\mathrm{e}}}} \tag{5.13}$$

It should be clear that the previous considerations are also applicable to the motion of our moon around the earth and the motion of the planets around the sun.

Example 5.9 If one want to place a communication satellite into a circular orbit of radius 6800 km. What must be its speed, and its period?

Solution From Equation (5.11) we obtain

$$v = \sqrt{\frac{GM_e}{r}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{6.8\times10^{6}}}$$

$$=7.66\times10^{3} \text{ m/s.}$$

The period is, from Equation (5.13),

$$t = \frac{2pr^{3/2}}{\sqrt{GM_e}} = \frac{2p\,(6.8\times10^{6})^{3/2}}{\sqrt{6.67\times10^{-11}\times5.98\times10^{24}}}$$

$$=1.55 \text{ hr.}$$

5.6 KEPLER'S LAWS

The fact that the planets move about the sun in such a way that the areal velocities are constant was found by Johannes Kepler in 1609. Keplere studied the data of his teacher Tycho Brahe and eventually formulated the following three laws applied to the solar system.

1. The law of orbits: Each planet moves in an ellipse with the sun as a focus.
2. The law of areas: The radius vector drawn from the sun to any planet sweeps out equal areas in equal times.
3. The law of periods: The square of the period of revolution of any planet is proportional to the cube of the major axis of the orbit.

PROBLEMS

5.1 A 2-kg mass moves in a circle with a speed of 4 m/s. If the radius of the circle is 0.5 m, what is the centripetal force acting on the mass?

5.2 A 2-kg mass is attached to a light string rotates in circular motion on a horizontal, frictionless table. The radius of the circle is 1.0 m, and the string can support a maximum force of 240 N. What is the maximum speed the mass can have before the string breaks?

5.3 A car rounds an unbanked curve with a radius of 50 m. If the coefficient of static friction between the tires and the road is 0.6, what is the maximum speed the car can have in order not to slide during the rounding.

5.4 A 200-g mass on a frictionless table is attached to a hanging block of mass 800 g by a cord through a hole in the table as in Figure 5.7. The suspended block remains in equilibrium while the mass revolves on the surface of the table in a circle of radius 0.6 m.

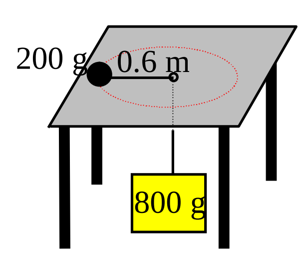


Figure 5.7 Problem 5.4.

a) What is the tension in the cord?
b) What is the speed of the mass?
c) What is the centripetal force acting on the mass?

5.5 A car moving at 50 km/hr want to turn a 15^{o}-banked curve with radius of 40 m on a rainy day (friction is neglected). Would the car make the turn successfully? If not with what speed must it move?

5.6 A small mass is placed 0.5 m from the center of a rotating, horizontal table that rotates with a constant speed of 1.5 m/s. The mass is in the verge of sliiping with respect to the turntable. What is the coefficient of static friction between the mass and the table?

5.7 A car travels over a hill, which can be regarded as an arc of a circle of radius 150 m, as in Figure 5.8. What is the maximum speed the car can have without leaving the road at the top of the hill?

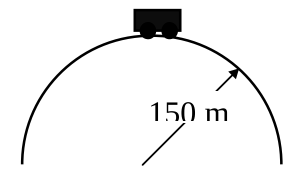


Figure 5.8 Problem 5.7.

5.8 A coin is placed inside an open basket, which in turn allowed to rotate in a vertical circle of radius 1.2 m. What is the minimum speed of the basket at the top of the circle if the coin not to fall off?

5.9 A 30-kg child sits in a conventional swing of length 2.5 m. The tension in each chain that support the seat of the swing at the lowest point is 300 N, and the mass of the seat is 4 kg.
a) What is the child's speed at the lowest point?
b) What is the force acting on the child by the seat?

5.10 A 0.2-kg pendulum bob passes through the lowest point with a speed of 6 m/s. What is the tension in the cord of the pendulum if it is 1 m long?

5.11 A small ball of mass m is suspended from a string of length L that makes an angle θ with the vertical. The ball revolves in a horizontal circle with constant speed (conical pendulum), as in Figure 5.9. Find the speed of the ball.

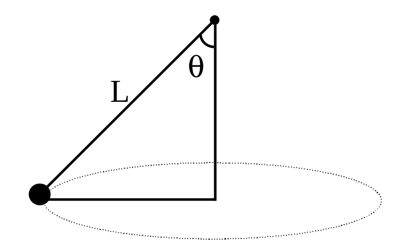


Figure 5.9 Problem 5.11.

5.12 An object tied to the end of a string is whirled in a vertical circle of radius *R*. What is the minimum speed below which the string would become loose at the highest point?

5.13 A Ferris wheel (أرجوحة الدولاب) with radius 20 m rotates at 8 m/s. Find the apparent weight of a 40-kg boy at
a) the top of the Ferris wheel,
b) the bottom of wheel.

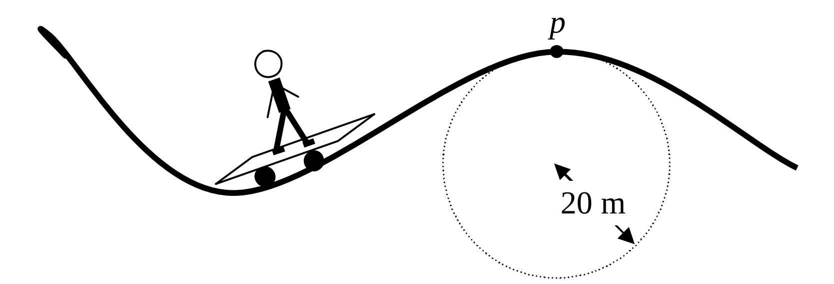


Figure 5.10 Problem 5.14.

5.14 A skater moves on an irregular track as shown in Figure 5.10. Point p in the track is at the top of an arc of a circle of radius 20m. What is the maximum speed of the skater at point *p* to remain in the track?

5.15 A person enters a Rotor of radius 3 m, as shown in Figure 5.11. If the coefficient of static friction between the person and the wall is 0.4, what is the minimum speed with which the Rotor must rotate such that the person is safe from falling?

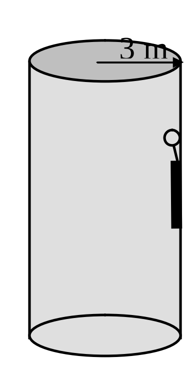


Figure 5.11 Problem 5.15.

5.16 A small block of mass m is placed inside a cone that is rotating about its axis, as in Figure 5.12. If the inside wall of the cone is frictionless, what is the speed of the cone to keep the mass from sliding down?

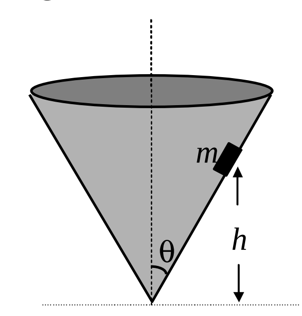


Figure 5.12 Problem 5.16.

5.17 A 3-kg mass is connected to a vertical rod by means of two massless strings, as in Figure 5.13. The strings are taut, and form two sides of equilateral triangle of length 2 m. The rod rotates about its axis and the mass rotates in a horizontal plane. If the tension in the upper string is 120 N,

a) draw the free-body diagram of the mass,

b) calculate the tension in the lower string,

c) calculate the speed of the mass.

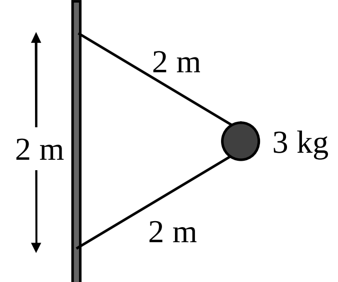


Figure 5.13 Problem 5.17.

5.18 A distance of 0.5 m. separates two particles of mass 200 kg, and 500 kg What is the gravitational force

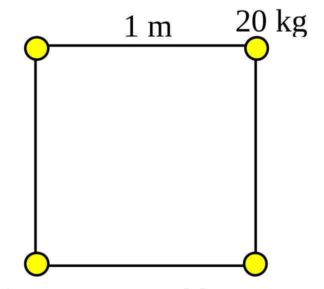


Figure 5.14 Problem 5.19.

exerted by one particle on the other?

5.19 Four identical balls each of mass 20 kg are located at the corners of a square of side 1 m, as in Figure 5.13. Calculate the total force acting on one ball from the other three balls.

5.20 What would be the weight of a 90-kg man at the top of a hill of height 500 km?

5.21 A satellite of mass 400 kg is in a circular orbit of radius 6.1×10^6 m about the earth. Calculate

a) The period of its revolution,

b) The gravitational force acting on it.

CHAPTER 6

WORK AND ENERGY

6.1 WORK

Consider an object displaced a distance **s** under the action of the constant force **F** as shown in Figure 6.1. The work done by this force is defined as the product of the magnitude of the displacement and the component of the force in the direction of the displacement. Since the component of **F** in the direction of **s** is $F\cos\theta$, the work W done by F is given by

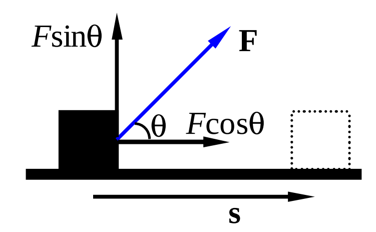


Figure 6.1 A constant force **F** acts on an object and displaced it a distance **s**.

$$W = (F\cos\theta)s \qquad 6.1$$

Comparing Equation 6.1 with definition of the scalar product the of two vectors, we conclude that the work done by a constant force can be expressed as

$$W = \mathbf{F}\cdot\mathbf{S} \qquad 6.2$$

From this definition, we see that work is done by **F** on an object if the object undergoes a displacement and **F** has a nonzero component in the direction of **s**. For example, if you push hard against a wall, no work is done by you on the wall even though you tire. Also the work done by a force is zero when the force is perpendicular to the displacement. Thus, we find that the meaning of work in physics is different from its common meaning in day-to-day affairs.

The work may be positive, or negative depending on the direction of **F** relative to **s**. It is positive if $F\cos\theta$ is in the direction of **s**, and is negative if $F\cos\theta$ is in the opposite direction of **s**.

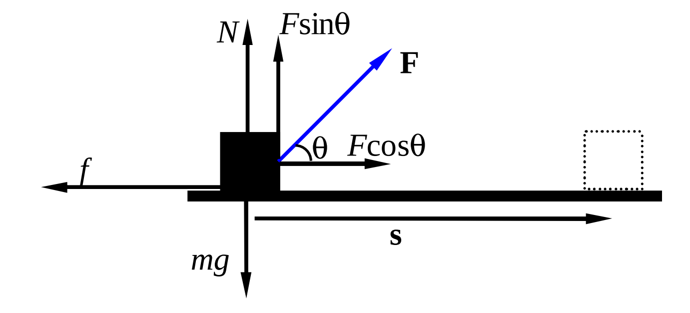


Figure 6.2 The work done by a frictional force is always negative.

An example of the negative work is the work done by a frictional force as a body slides along a rough surface (Figure 6.2). Since the angle q between the frictional force $\boldsymbol{f}$ and the displacement $\mathbf{s}$ is $q = p$, the work done by this frictional force is given by

$$W_f = \boldsymbol{f} \cdot \mathbf{s} = f\, s \cos p = -\, f\, s \qquad 6.3$$

It is clear from Figure 6.2 that the normal force N and the weight mg do no works since these two force are perpendicular to the displacement, that is $q = p/2$.

> **Remark:** Since the frictional force is in opposite direction with the displacement, (Figure 6.2), the work done by any frictional force is always negative.

From Equation 6.2 we conclude that the work is a scalar quantity. Its unit is force multiplied by length. Therefore, the SI units of work is Newton meter (N.m) or Joule (J), while the cgs unit is erg.

Example 6.1 A block of mass 2 kg moves under the influence of a force F= 20 N, which makes an angle of 37° above the horizontal. The block moved a distance $s = 4\,\mathrm{m}$, on a rough surface of $\mu = 0.2$, as shown in Figure 6.3. Calculate,

a) the work done by F

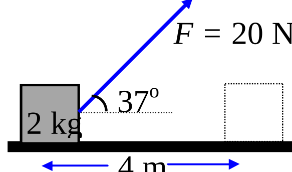


Figure 6.3 Example 6.1.

b) the work done by friction.
c) the net work done on the block.

Solution a) The work done by the force *F* is, from Equation 6.2

$$W_F = Fs\cos q$$
$$= (20)(4)(\cos 37^o) = 63.9 \text{ J.}$$

b) Using equation 6.3, the work done by the force of friction is

$$W_f = -f\, s = -\mu N s$$

but

$$N = mg - F\sin\theta \text{ (why?)}$$

so

$$W_f = -0.2 \times (19.6 - 12.0) \times 4 = -6.1\text{J}$$

c) Since the weight (*mg*) and the normal force (*N*) do not do any work (why?), the net work W_{net} is then

$$W_{net} = W_F + W_f = 63.9 - 6.1 = 57.8 \text{ J.}$$

6.2 WORK DONE BY A VARYING FORCE

Consider an object being displaced along the *x*-axis under the action of a force that varies with respect to the position *x*. To find the work done by such a force as the object moves from an initial point x_i to a final point x_f, we cannot use Equation 6.2 because it applies only for a constant force. To manage such a situation we devide the displacement into small intervals Δx such that the force can be considered constant over such intervals. The work done by the force over this small displacement can be expressed as

$$\Delta W = F_x \Delta x \tag{6.4}$$

The total work done along the displacement from x_i to x_f is the sum of many such terms:

$$W = \sum \Delta W = \sum_{x_i}^{x_f} F_x \Delta x \tag{6.5}$$

If the displacement intervals are allowed to approach zero, then the number of terms in the sum increases infinitely. This limit of the sum is called the integral and is represented by

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx \tag{6.6}$$

In general we can express the work done by any force for the displacement from initial point *i* to a final point *f* as

$$W = \int_i^f \mathbf{F} \cdot d\mathbf{s} \tag{6.7}$$

Note that if the force **F** is constant over the displacement, we recover Equation 6.2.

6.3 Work done by a spring

As an example of force varies with position we consider here the force of a spring. Figure 6.4(a) shows a spring with one end is fixed, while the other end is attached to a block. The spring is in its equilibrium state, that is, neither compressed nor extended. In Figure 6.4(b) the block is displaced to the right and the spring is now stretched a distance *x*. If the block is displaced to the left as in

Figure 6.4(c) the spring is now compressed. Again the spring will exert a force on the body toward its equilibrium position. The magnitude of the force in both cases is given by **Hook's law:**

$$F_s = -kx \qquad (6.8)$$

where x is the displacement of the body from its equilibrium position (x=0). The force constant k is a measure of the stiffness of the spring. The minus sign in Equation (6.8) tells that the force of the spring is always opposes the displacement. Let us calculate the work done by the force F_S , as the body moves from an initial position x_i to a final position x_f . Applying Equation (6.7), we get

Figure 6.4 (a) The block is at the equilibrium position and no force exerted by the spring. (b) The spring is stretched and the of the spring force opposes the displacement. (c) The spring is compressed and again the force opposes the displacement.

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{x_i}^{x_f} -kx dx = -k\int_{x_i}^{x_f} x dx$$

,
or

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 . \qquad (6.9)$$

The work done by a spring, W_s, can be positive or negative depending on whether the mass moves toward or a way from the equilibrium position:

1- If the mass moves toward the equilibrium position then $x_f = 0$ and, from Equation (6.9), the work is positive. This is due to the fact that the force and the displacement, in this case, are in the same direction.
2- If the mass moves a way from the equilibrium position then $x_i = 0$, and the work is negative. Here the force and the displacement are in opposite directions.

The work done by an external force in compressing or stretching a spring is equal to the negative of the work done by the spring's force during the corresponding displacement.

Example 6.2: A block is tied to a spring with force constant of 80 N/m as shown in Figure 6.4. The spring is compressed a distance 3 cm from equilibrium position.
a) Calculate the work done by the spring as the block moves from its equilibrium to its compressed position.
b) Calculate the work done by the spring as the block returns to its equilibrium position.

Solution: a) The block was at its equilibrium position $x_i = 0$ and moves to a final position $x_f = -3\,\text{cm} = -0.03\,\text{m}$. The work done during this interval is, from Equation 6.9 as

$$W_s = 0 - \frac{1}{2}\times 80\times(-3\times10^{-2})^2 = -3.6\times10^{-2}\,\text{J}$$

b) Now $x_i = -3\,\text{cm} = -0.03\,\text{m}$ and $x_f = 0$. So from Equation 6.9 we have

$$W_s = \frac{1}{2}\times 80\times(-3\times10^{-2})^2 - 0 = 3.6\times10^{-2}\,\text{J}.$$

6.4 WORK-KINETIC ENERGY THEOREM

Let us consider that the net force acting on an object is in x direction, then equation (6.7) takes the form

$$W_{net} = \int_i^f F_x \, dx \tag{6.10}$$

where F_x is the net force acting on the body in the x direction. Newton's second law states that $F_x = ma_x$, and the acceleration a can be expressed as

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$$

Substituting this in Equation (6.6), we have

$$\begin{aligned} W_{net} &= \int_i^f mv\frac{dv}{dx}dx = \int_{v_i}^{v_f} mv\,dv = \frac{1}{2}mv^2\Big|_{v_i}^{v_f} \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \end{aligned} \tag{6.11}$$

The quantity $K = \frac{1}{2}mv^2$ is called the **kinetic energy** of a particle of mass m and speed v. It is convenient to write Equation 6.11 as

$$W_{net} = K_{\mathrm{f}} - K_{\mathrm{i}} = \Delta K \tag{6.12}$$

That is, the net work done by the force F_x along x-direction from $x = x_i$, to $x = x_f$ is given by the change in kinetic energy, ΔK.

Example 6.3: Consider the problem of example (6.1),. If the initial velocity is zero, find the final velocity after the block moves a distance 4 m.

Solution: From example (6.1) the net work is given by

$$W_{net} = 57.8\,\mathrm{J}\ .$$

Applying the work-energy theorem with v_i=0, we have

$$W_{\text{net}} = \frac{1}{2} m v_f^2$$

or

$$v_f^2 = \frac{2W_{net}}{m} = \frac{2\times 57.8}{2} = 57.8\,\mathrm{m^2/s^2}\ ,$$

so

$$v_f = 7.6\ \mathrm{m/s}.$$

Example 6.4 A mass of 2 kg is pushed up a rough inclined plane by a force F=20 N. The mass is displaced a distance 0.5 m on the inclined plane. Calculate,

a) the work done by the force of gravity,

b) the work done by the force F=20 N,

c) the work done by the force of friction if μ_k=0.2,

d) If the mass has a kinetic energy of 1.2 J at the beginning of the displacement, what is the kinetic energy at the end of the displacement?

Solution: a) Since the force of gravity is downward, the work done by the force of gravity is given by

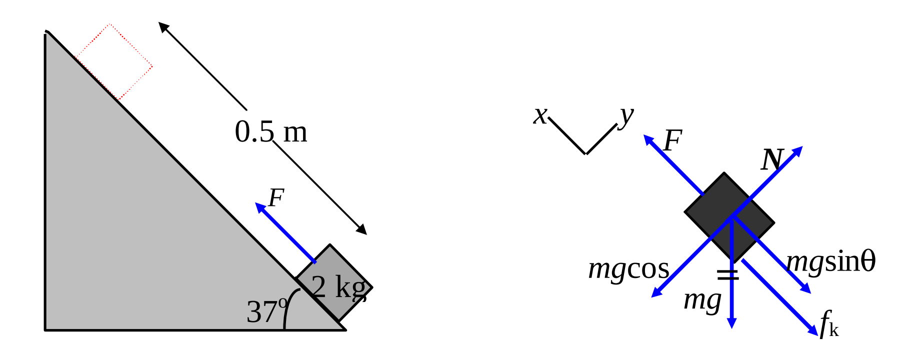


Figure 6.5 Example 6.4

$$W_g = -(mg\sin 37^o)d = -(2\times 9.8\times 0.6)\frac{1}{2} = -5.88J$$

b) **F** is in the same direction as the displacement **s**, so the work done by **F** is,

$$W_F = Fs = 20\times 0.5 = 10 \text{ J}.$$

c) The force of friction is given by

$$f = mN = mmg\cos 37^o = 0.2\times 2\times 9.8\times \cos 37^o = 3.13\text{N}$$

Now the work done by this frictional force is

$$W_f = -fs = -3.13\times 0.5 = -1.57 \text{ J}$$

d) Using the work-kinetic energy theorem we have

$$\Delta K = W_{net} = W_g + W_F + W_f$$
$$= -5.88 + 10 - 1.57 = 2.55 \text{ J}.$$

But $\Delta K = K_f - K_i$. So

$$K_f = \Delta K + K_i = 2.55+1.2=3.75 \text{ J}$$

6.5 POWER

The power is defined as time rate at which work is done. **The average power** during the time interval Δt is defined as

$$\overline{P} = \frac{\Delta W}{\Delta t} \qquad (6.13)$$

The instantaneous power is the limiting value of $\overline{P}$, that is

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad (6.14)$$

If **F** is constant, then dW=**F**.d**s**, and the power (6.14) becomes

$$P = \frac{dW}{dt} = \mathbf{F}.\frac{d\mathbf{s}}{dt} = \mathbf{F}.\mathbf{v}\ , \qquad (6.15)$$

where the velocity **v** is defined by $\frac{d\mathbf{s}}{dt}$. The unit of power is J/s or watt, (W), with 1W=1 J/s=1 kg.m^2/s^3.

Example 6.5: A 1500-kg car accelerates uniformly from rest to a speed of 10 m/s in 3 s. Find
a) the work done on the car in this time,
b) the average power delivered by the engine in the first 3 s,
c) the instantaneous power delivered by the engine at t=2 s.

Solution: After 3 s, v_f=10 m/s, m=1500kg

a) the work done is given by

$$W = \frac{1}{2}mu_f^2 - 0$$
$$= \frac{1}{2}\times 1500\times(10)^2 = 7.50\times 10^4\,\text{J},$$

b) $$\overline{P} = \frac{7.5\times 10^4}{3} = 2.5\times 10^4\ \text{W},$$

c) let us find the acceleration *a*, from $v = v_o + at$, we have,

$$a = \frac{v - v_o}{t} = \frac{10 - 0}{3} = 3.33\,\text{m/s}^2.$$

The velocity at t=2s, is then

v=0+3.33×2=6.66 m/s

and the force of the engine is calculated using Newton's second law

F=ma=1500×3.33=5000 N.

Now, the instantaneous power is

P=Fv=5000×6.66=3.33×10^4 W.

PROBLEMS

6.1 A block is pushed 10 m along a horizontal surface by a 3-N horizontal force. The frictional force on the block is 1.5 N.
a) How much work is done by the 3-N force?
b) How much work is done by the frictional force?

6.2 A man pushes a 10-kg block 10 m, along a rough, horizontal surface with a 40-N force directed 37° below the horizontal. If the coefficient of kinetic friction is 0.2, calculate the total work done on the block.

6.3 A block of mass 20-kg is pushed by a horizontal force of 60 N on a rough horizontal surface a distance 4 m. If the block moves with constant speed, calculate
a) the work done by the 60 N force,
b) the work done by a frictional force,
c) the coefficient of kinetic friction

6.4 A particle of mass 0.5 kg is released from rest at point *A* inside a rough hemi-spherical bowl of radius 1 m, as shown in Figure 6.6. If the particle can reach a maximum height of 0.8 m (point *B*), calculate
a) the work done by gravity,
b) The work done by friction.

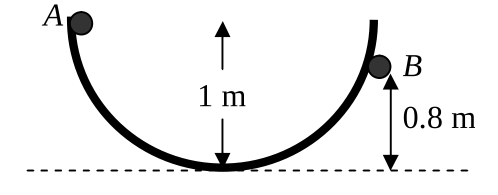


Figure 6.6 Problem 6.4.

6.5 A block of mass m = 12 kg is drawn at constant speed a distance s=20 m along a horizontal floor by a rope exerting a

constant force F=40 N making an angle of 53° above the horizontal. Compute,
a) the total work done on the block
b) the work done by the rope on the block
c) the work done by friction on the block
d) the coefficient of kinetic friction between block and the floor.

6.6 A block of mass 50 kg is attached to a cord that is wrapped around a fixed pulley as shown in Figure 6.7. The block is lowered at constant acceleration of 2.5 m/s^2. When the block has fallen a distance 4 m, find
a) The work done by the cord,
b) the work done by the weight of the block.

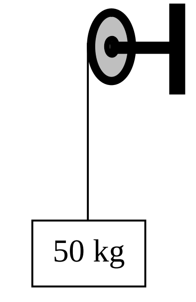


Figure 6.7 Problem 6.6.

6.7 A Force $F_x = (10x + 2)$ N, where x is in m, is acting on a body from x=0 to x = 2 m. Find the net work done by this force as the body moves from x =0 to x = 2 m.

6.8 A block of mass 30 kg is hung vertically on a light spring with spring constant k. After permanently coming to rest, the spring stretches a distance of x = 10 cm.
a) What is the work done by gravity during this motion?
b) What is the work done by the spring during this motion?
c) What is the net work done on the block?
d) Is the answer of (c) is surprising?

6.9 A force $\mathbf{F} = (-5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ N acts on a particle which is displaced $\mathbf{s} = (-2\mathbf{i} - 2\mathbf{k})$ m.
a) Find the work done by the force on the particle.

b) Find the angle between **F** and **s**.

6.10 A force vector **F** is 2 N in magnitude, and points in the positive y direction. The displacement vector **s** has a negative x-component of 5 m, a positive y-component of 3 m, and no z-component. Find,
a) the work done by the force **F**,
b) the angle between **F** and **s**.

6.11 A small ball of mass 1 kg is attached to a light, 2 m long string. The ball is released from the horizontal position A, as shown in Figure 6.7.
a) Find the work done by gravity as the particle moves from point A to point B, the lowest position of the ball.
b) Find the speed of the particle at point B.

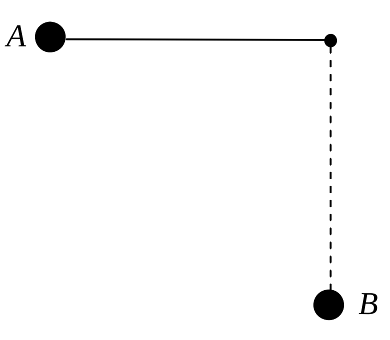
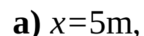


Figure 6.7 Problem 6.11.

6.12 A car of mass 2000 kg is pushed from rest to a speed v, a distance 20 m. The work done on the car is 6000 J, find
a) the final speed v of the car,
b) the horizontal force exerted on the car, assuming it is constant.

6.13 A 4-kg mass has an initial velocity $\mathbf{v} = (-3\mathbf{i} - \mathbf{j})$ m/s.
a) Find the kinetic energy at this time.
b) Find the change in its kinetic energy if its velocity changes to (4**i**+2**j**) m/s.

6.14 A 5-kg body is subject to a force that varies with position as shown in Figure 6.8. The body starts from rest at x=0. Find the speed of the body at



a) x=5m,
b) x=10m,
c) x=15m.

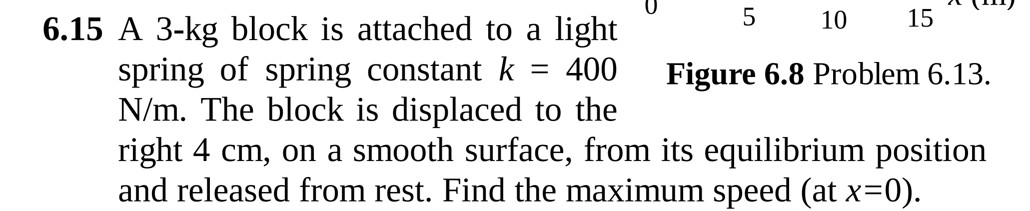
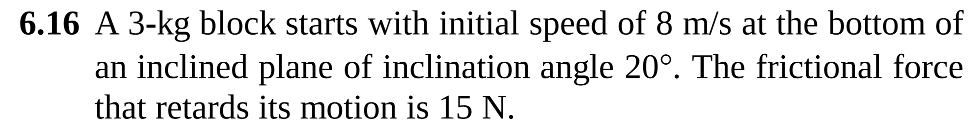


Figure 6.8 Problem 6.13.

6.15 A 3-kg block is attached to a light spring of spring constant k = 400 N/m. The block is displaced to the right 4 cm, on a smooth surface, from its equilibrium position and released from rest. Find the maximum speed (at x=0).



6.16 A 3-kg block starts with initial speed of 8 m/s at the bottom of an inclined plane of inclination angle 20°. The frictional force that retards its motion is 15 N.
a) If the block is directed up the incline, how far will it move before it stop?
b) Will it slide back down the incline?

6.17 A car of mass 1000 kg is pulled along a rough surface with coefficient of friction μ_k = 0.3.
a) How much power must the engine deliver to move the car at constant speed of 40 m/s.
b) How much work does the engine in 2 minutes do?

6.18 A car engine delivers a power of 3×10^4 W when moving with constant speed of 20 m/s. Find the resistive force acting on the car at this speed.

6.19 A boat moves at a constant speed of 20 m/s. If the resistive force of the water is 10 N, how much power is produced by the motor?

6.20 A small block of mass 0.25 kg is pushed againsed a spring of force constant 800 N/m, compressing it a distance of o.1

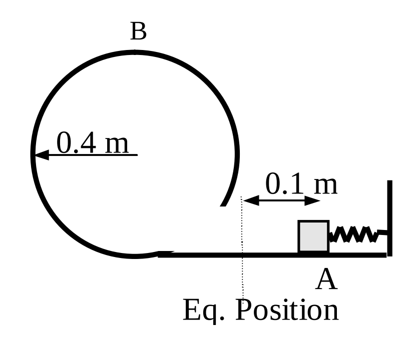


Figure 6.9 Problem 6.20.

m. When released, the block travels along a smooth circular track of radius 0.4 m, as shown in Figure 6.9.
a) Calculate the net work acting on the block in going from point A to point B (the top of the track).
b) Find the force the track exerts on the block at point B.

CHAPTER 7

CONSERVATION OF ENERGY

7.1 CONSERVATIVE AND NON-CONSERVATIVE FORCES

conservative Forces The force is conservative if the work done by it on a particle that moves between two points depends only on these points and not on the path followed. This means that in moving a particle, by a conservative force, from point *A* to point *B*, (Figure 7.1) the work done along path 1 = work done along path 2. As an example of a conservative force is the force of the spring. The work done by such a force is, from Equation 6.9,

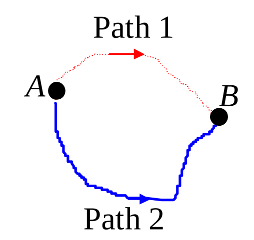


Figure 7.1 The work done, by a conservative force, in moving a particle from point *A* to point *B* does not depend on the path followed.

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

As it is clear from the Equation, the work done by a spring depends only on x_i and x_f (the initial and final positions).

The force due to gravity is another example of a conservative force. Consider a particle goes from point *A* to a point *B* that is higher than A by a distance H, as shown in Figure 7.2. The work done by gravity if the particle followed path 1, (the solid path) is simply $W_1 = -mgH$. If the particle followed another path, say path 2, (the dashed path)

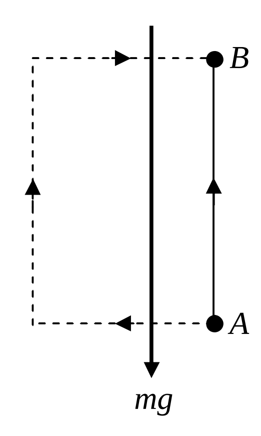


Figure 7.2 A particle goes from point *A* to point *B* through two different paths. The solid path, path1 and the dashed path, path2.

we see again that the work done by gravity is $W_2 = -mgH$, that is the work is independent of the path followed.

Non-conservative Forces The force is non-conservative if the work done by that force on a particle that moves between two points depends on the path taken between those points, i.e.,

The work done along path 1 ≠ work done along path 2. The Frictional force is an example of a non-conservative force. Suppose you were displaced a book between two points on a rough, horizontal surface such as a table, as shown in Figure 7.3. The work done by a friction force is -*fd*, where *d* is the distance between the two points *A* and *B*. It is clear that the work done along path 1 is greater than the work done along path 2, since path 1 is greater than path 2.

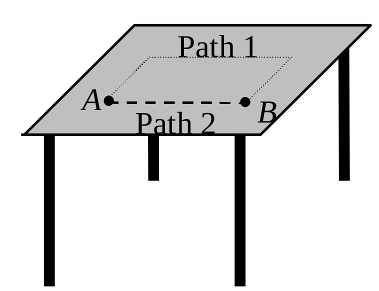


Figure 7.3 A particle moves from point *A* to point *B* along a rough table. The work done by the frictional force along the path 1, (The solid path) is greater than the work done along path 2, (the dashed path).

7.2 POTENTIAL ENERGY

If an object is thrown upward, the work W_g done on the object by its weight is negative, that is, energy is transferred from the kinetic energy to the weight. In another ward, energy is transferred from the kinetic energy to another type of energy called the gravitational **potential energy *U***. Now if the object begins to fall down back the transfer of the energy is reversed. The work done by the weight of the object W_g is now positive and energy is transferred from the potential energy to the kinetic energy.

For either rise or fall, the change in the gravitational potential energy ΔU is defined to equal the negative of the work done by the weight, that is

$$\Delta U = -W \tag{7.1}$$

The same relation holds if an object attached to a spring is moving compressing, or stretching, the spring and then returning back to its equilibrium position.
To generalize we say that for every conservative force we associate a potential energy such

$$W_c = \int_i^f \boldsymbol{F} \cdot d\boldsymbol{s} = -\Delta U \tag{7.2}$$

If the force is one dimensional, say along the *x*-axis, the work done by such a force acting on an object as the object moves through a distance Δx is $F(x)\Delta x$. From Equation 7.1 we can write

$$\Delta U(x) = -W = -F(x)\Delta x$$

Letting Δx approaches zero $(\Delta x \to 0)$ we obtain for the force

$$F(x) = -\frac{dU(x)}{dx}. \tag{7.3}$$

Note that Equation 7.3 holds only for one dimensional motion. For two or three dimensional motion, the formula is more complicated and is out of the scope of this course.

7.3 GRAVITATIONAL POTENTIAL ENERGY

Since the force of gravity is conservative, we can associate a gravitational potential energy function U_g to this force. To calculate such potential energy, we consider a particle that makes a displacement from an initial point i to a final point f, as shown in Figure 7.4. The force of gravity does a negative work given by $-mgy$ (why negative?). Now from Equation 7.1 we conclude

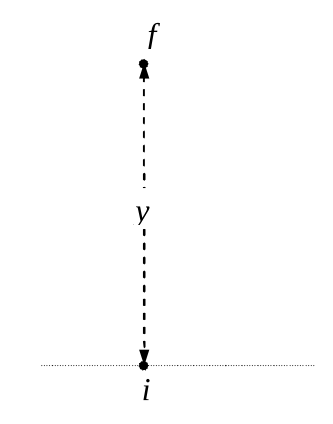


Figure 7.4 A particle displaced a distance y from point i to point f.

$$\Delta U_g = mgy,$$

or

$$U_f - U_i = mgy$$

Since we are concerned only by the change in the potential energy, U_i can be chosen to be zero. The horizontal level at which U_i is zero is called the level of zero potential energy. The gravitational potential energy at a point is then defined as

$$U_g = mgy, \qquad (7.4)$$

where y is the vertical displacement above an arbitrary horizontal level (level of zero potential energy).

Remark: The gravitational potential energy is positive if the body is above the level of zero potential energy, and negative if the body is below the level.

Example 7.1: A book of mass 1.2 kg is on a horizontal table that is 1.5 m high. The book and the table are in a room of height 2.8 m.
a) What is the gravitational potential energy of the book if the level of zero potential energy is taken to be (i) at the table's surface, (ii) at the floor, and (iii) at the ceiling?
b) When the book drops to the floor, calculate ΔU for the three choices of the zero potential energy.

Solution *a*) Using Equation 7.4 we have
(i) y=0 if the level of zero potential energy is taken to be the surface of the table and this leads to $U_g = 0$
(ii) Now if the level of zero potential energy is taken to be the floor, we have y=1.5 m and this gives

$$U_g = mgy = (1.2)(9.8)(1.5) = 17.67\,\mathrm{J}\,.$$

(iii) $y = -(2.8 - 1.5) = -1.3\,\mathrm{m}$ when the level of zero potential energy is taken to be at the ceiling. Thus we obtain

$$U_g = (1.2)(9.8)(-1.3) = -15.29\,\mathrm{J}$$

b) $\Delta U_g = mg(y_f - y_i) = mg\Delta y$. Now for the three choices we have (i) $\Delta y = -1.5 - 0 = -1.5\,\mathrm{m}$,
(ii) $\Delta y = 0 - 1.5 = -.15\,\mathrm{m}$, and
(ii) $\Delta y = -2.8 - (-1.3) = -.15\,\mathrm{m}$.

So for all the choices we obtain the same result

$$\Delta U_g = (1.2)(9.8)(-1.5) = -17.64\,\mathrm{J}$$

7.4 POTENTIAL ENERGY OF A SPRING

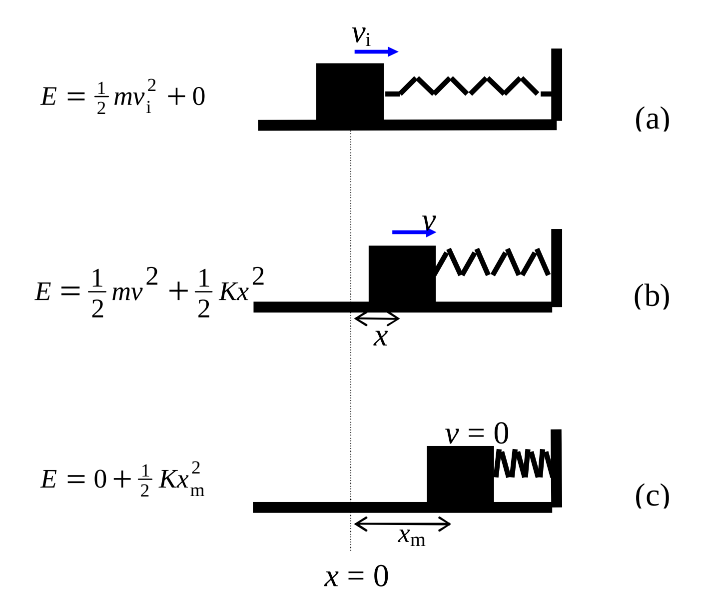


Figure 7.5 A block, attached to a spring, moves along a horizontal, frictionless surface. (a) The block is at the equilibrium position where the potential energy is zero and the mechanical energy is all kinetic. (b) The mechanical energy is the sum of the kinetic energy and the potential energy. (c) The kinetic energy is totally transferred, and stored as potential energy in the spring, so the mechanical energy is all potential.

Consider a system consists of a spring with spring constant k, and a mass m that slides on a frictionless surface, as in Figure 7.5. Since the force of the spring is conservative force, a potential energy U_s can be associated with this force. As the particle goes from the equilibrium position to a position where the spring is compressed or stretched, the force of spring does a work given by

$$W_s = -\tfrac{1}{2}kx^2$$

The work is negative because the force and the displacement are opposite in both cases. Again using Equation 7.1 we can calculate the potential energy stored in a spring (elastic potential energy), as

$$U_s = \tfrac{1}{2}kx^2, \qquad (7.5)$$

where x is the amount of compressing, or stretching the spring. Here the potential energy at the equilibrium position is taken to be zero. The conservation of mechanical energy of the mass-spring system can be written as

$$E_i = E_f$$

or

$$\tfrac{1}{2}mv_{\mathrm{i}}^2 + \tfrac{1}{2}kx_{\mathrm{i}}^2 = \tfrac{1}{2}mv_{\mathrm{f}}^2 + \tfrac{1}{2}kx_{\mathrm{f}}^2 \qquad (7.6)$$

REMARKS:

- The potential energy of the spring is zero at the equilibrium point (unstretched position, $x = 0$), and the total energy is $E = \tfrac{1}{2}mv_i^2$ = maximum value of kinetic energy (v is maximum)
- The potential energy is a maximum when x is a maximum value(when $v = 0$, at maximum compression)
- U_s is always positive, since x^2 is always positive.

7.5 CONSERVATION OF MECHANICAL ENERGY

Suppose a particle moves under the influence of a conservative force F. Then the work-energy theorem from chapter 6 tells us that the work done by that force equals the change in kinetic energy:

$$W_C = \Delta K$$

Since the force is conservative, we have

$$W_C = -\Delta U = \Delta K$$

or

$$\Delta K + \Delta U = 0 \qquad (7.7)$$

The law of conservation of mechanical energy states that the change in mechanical energy is always zero. In other words, if the kinetic energy of a conservative system increases (or decreases) by some amount, the potential energy must decreases (or increases) by the same amount. Equation (7.7) can be written as

$$K_i + U_i = K_f + U_f ,$$

or

$$E_i = E_f , \qquad (7.8)$$

where the **mechanical energy** E is defined as

$$E = K + U \qquad (7.9)$$

Example 7.2: A small block of mass m = 2 kg is released from a height of h = 10 m above the ground as shown in Figure 7.6. Using the law of conservation energy determine,
a) the speed of the block at an altitude of y = 4 m above the ground,
b) the velocity of the ball just before it hits the ground.

Solution: a) Applying the conservation of energy principle between points *A* and *B*, we get

$$K_A + U_A = K_B + U_B$$

$$0 + mgh = \tfrac{1}{2}mv_B^2 + mgy$$

or

$$v_B = \sqrt{2g(h-y)} = \sqrt{19.6(10-4)}$$
$$= 10.8\text{m/s}$$

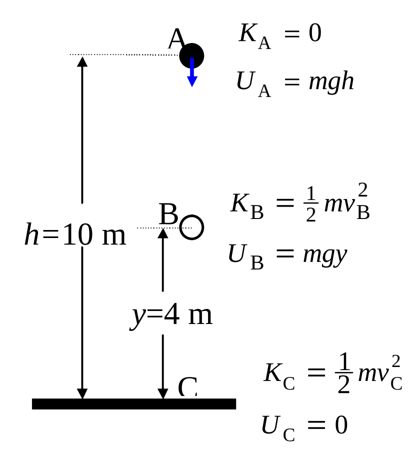


Figure 7.6 Example 7.1

b) Conservation of energy between points A and C gives

$$K_A + U_A = K_C + U_C$$

$$0 + mgh = \tfrac{1}{2}mv_C^2 + 0$$

or

$$v_C = \sqrt{2gh} = \sqrt{19.6 \times 10} = 14\text{ m/s.}$$

7.6 NONCONSERVATIVE FORCES AND WORK-ENERGY THEOREM

Suppose that there are conservative and non-conservative forces acting on a system, and these forces do work. The net work done on the system is then can be written as the sum of the work done by the conservative force W_C, and the work done by the non-conservative forces W_{NC}, that is

$$W_{net} = W_C + W_{NC} \qquad (7.10)$$

Substituting for W_{net} from Equation 6.12 and for W_C from Equation 7.2, Equation 7.10 becomes

$$\Delta K = -\Delta U + W_{NC}$$

Or

$$\Delta K + \Delta U = W_{NC} \qquad (7.11)$$

Since the mechanical energy E is given by $E = K + U$, we can express Equation 7.11 as

$$W_{NC} = \left(K_f + U_f\right) - \left(K_i + U_i\right) = E_f - E_i \qquad (7.12)$$

This means that the work done by all non-conservative forces equals the change in the mechanical energy of the system. Note that if W_{nc} is zero we recover Equation 7.8 as expected.

STRATEGY for solving problems using the conservation of energy principle.

(i) Select a horizontal level for the zero gravitational potential energy.
(ii) Define two points: one as an initial point and the other as a final point.
(iii) Find the potential energy and the kinetic energy at these two points.
(iv) If there is a spring, then the total potential energy of the system is $U = U_g + U_s$.
(v) If there are friction forces, then calculate W_{nc}. If not then $W_{nc} = 0$.
(vi) Now use Equation 7.7 or Equation 7.8 to find the unknowns.

Example 7.3: A 2-kg mass slides down a rough inclined plane, as shown in Figure 7.7. the mass starts from rest, and the friction force is given by f = 5 N.

a) Use energy method to find the speed of the block at the bottom of the incline.

b) If the inclined plane is frictionless find the speed at that point.

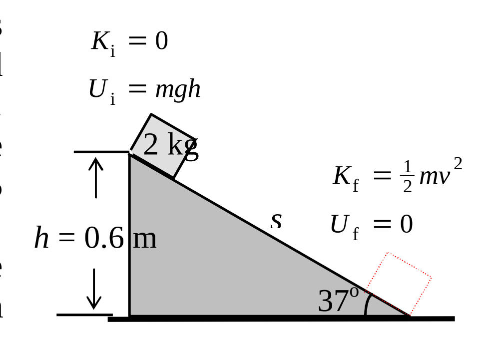


Figure 7.7 Example 7.2

Solution: The ground is chosen as the level for the zero potential energy, and the initial point is chosen at the top of the plane, while the final point is chosen at the bottom of the plane. Now

$$E_i = K_i + U_i = \tfrac{1}{2}mv_i^2 + mgy_i$$
$$= 0 + 2 \times 9.8 \times 0.6 = 11.76\text{J},$$

and

$$E_f = K_f + U_f = \tfrac{1}{2}mv_f^2 + mgy_f$$
$$= \tfrac{1}{2} \times 2 \times v^2 + 0 = v^2.$$

Since the plane is rough, we have

$$W_{nc} = -fs = -5 \times \frac{0.6}{\sin 37} = -4.98\text{J}.$$

Now applying Equation (7.12) we get

$$v^2 - 11.76 = -4.98,$$

or

$$v = \sqrt{6.78} = 2.6\text{m/s}.$$

b) In this case , $W_{nc} = 0$, and Equation 7.8 becomes

$$v^2 - 11.76 = 0,$$

or

$$v = \sqrt{11.76} = 3.43\text{m/s}.$$

Example 7.4: Two blocks are connected by a light string that passes over a frictionless pulley as shown in Figure 7.8. The mass m_1 lies on a rough surface, and the system is released from rest when the spring unsretched (x = 0). The mass m_2 falls a distance h before coming to rest. Calculate the coefficient of kinetic friction between m_1 and the surface.

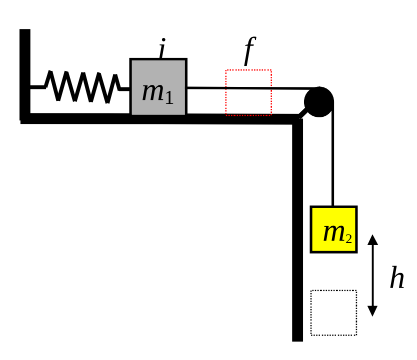


Figure 7.8 Example 7.3

Solution: In this example there are two forms of potential energy: the gravitational potential energy and the potential energy of the spring, so

$$\Delta U = \Delta U_g + \Delta U_s.$$

But ΔU_g is due only to m_2, therefore we have

$$\Delta U_g = -m_2 gh,$$

while ΔU_s is associated only to m_1, so

$$\Delta U_s = \tfrac{1}{2} kh^2.$$

Since the initial and the final speed of the system is zero, the $\Delta K = 0$.

The work done by the frictional force is given by

$$W_{\text{nc}} = -fh = -(\mu m_1 g)h .$$

Now applying Equation (7.11) we get

$$-\mu m_1 gh = 0 - m_2 gh + \tfrac{1}{2}kh^2 ,$$

or

$$\mu = \frac{m_2 g - \tfrac{1}{2}kh}{m_1 g} .$$

PROBLEMS

7.1 A block of mass 2 kg is at the top of an inclined plane of height 1.8 m and inclination angle of 35^o, as shown in Figure 7.9. Find the gravitational potential energy of the block if the level of zero potential energy is

(a) at the bottom of the plane,

(b) at the top of the plane.

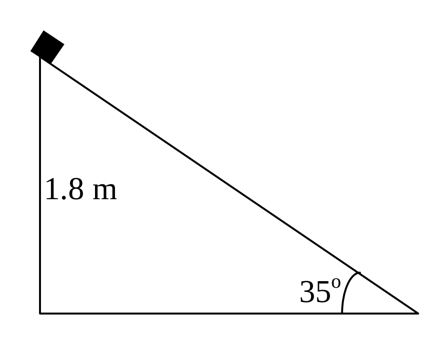


Figure 7.9 Problems 7.1, and 7.2.

7.2 Referring to the above problem, if the block drops to the bottom of the plane, find

a) The change in the gravitational potential energy for the two choices of the zero level of potential energy,

b) The work done by gravity.

7.3 A conservative force acting on a particle varies with x according to $\mathbf{F} = (-2x + 3x^2)\mathbf{i}$ N, with x is in meter.

a) Find the potential energy associated with this force, where U=0 at x=0.

b) Calculate the change in potential energy and change in kinetic energy as the particle displaced from x=2 to x=4.

7.4 Calculate the change in gravitational potential energy when a 850-kg elevator moves from the floor level to the top of a building that is 30 m high.

7.5 In Figure 7.10 the string has a length l = 0.5 m. When the ball is released, it will swing down the dotted arc. Find the speed of the ball when it reaches the lowest point in its swing.

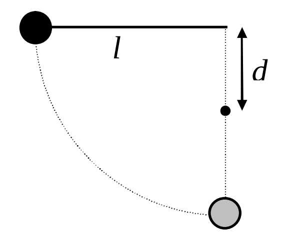


Figure 7.10 Problems 7.5 and 7.6.

7.6 If a nail is located a distance d below the point of suspension in Figure 7.10. Show that d must be at least $0.6l$ if the ball is to swing completely around in a circle centered on the nail.

7.7 The system shown in Figure 7.11 starts from rest. If the 4-kg block has fallen a distance 2 m, find the speed of the system by using conservation of energy.

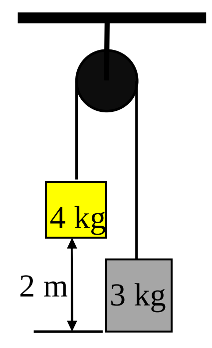




Figure 7.11 Problem 7.7.

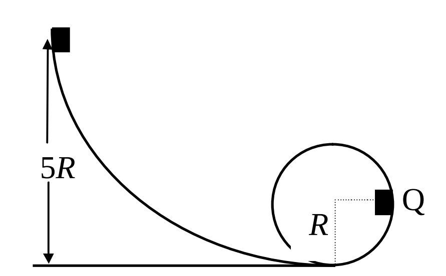


Figure 7.12 Problem 7.8.

7.8 A small block of mass m slides without friction around a loop-to-loop (Figure 7.12). If the mass is released from a height h= 5 R, find the speed of the block

a) at point Q,

b) at the top of the loop.

7.9 Two masses m_1=2 kg and m_2=4 kg are connected by a light rope passing over a frictionless pulley as shown in Figure 7.13. The mass m_2 is released from rest. Using the conservation of energy, find,

a) the velocity of mass m_1 just as m_2 hits the ground.

b) the maximum height to which m_1 will rise.

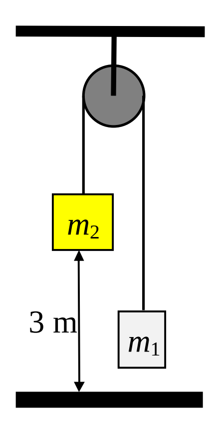


Figure 7.13 Problem 7. 9.

7.10 A pendulum of mass 5 kg and length 2 m has initial maximum speed of 4 m/s at the bottom. When the string makes an angle of 37° with the vertical, find

a)the change in the potential energy of the mass,

b) the speed of the mass,

c) the tension in the string,

d) the maximum height of the mass above the lowest position.

7.11 A block of mass 4 kg is pushed up a rough inclined plane with an initial speed of 6 m/s. The plane is inclined at an angle of 37° to the horizontal. The block comes to rest after traveling 2 m long.

a) Find the change in kinetic energy.

b) Find the change in potential energy.

c) Determine the frictional force on the block.

d) Find the coefficient of kinetic friction.

7.12 A small mass m=0.5 kg slides on an irregular path, starting from rest at point A as shown in Figure 7.14. The segment from A to B is frictionless, and the

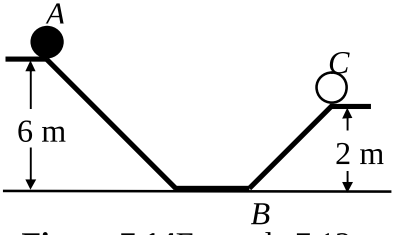


Figure 7.14 Example 7.12.

segment from B to C is rough.

a) Determine the speed of the mass at B.

b) If the mass comes to rest at C, find the total work done by friction in going from B to C.

c) What is the net work done by the non conservative forces as the mass moves from point A to point C?

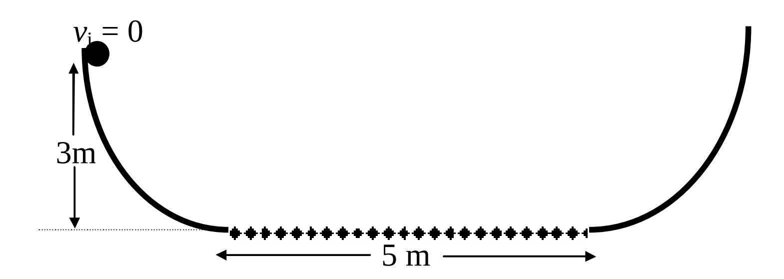


Figure 7.15 Problem 7.13.

7.13 A particle slides a long a curved track with flat part of length l = 5 m, as shown in Figure 7.15. The flat part is rough with $m_k = 0.3$, while the curved portions are smooth. The particle is released from a height of 3 m. Where dose the particle finally come to rest?

7.14 A particle of mass 100 g is released from rest at point A along the diameter on the inside of a smooth hemi-spherical bowl of radius R=9 cm, as in Figure 7.16.

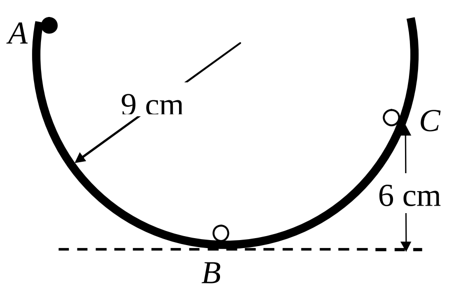


Figure 7.16 Problem 7.14.

a) Calculate the gravitational potential energy of the particle at point A relative to point B.

b) Find its kinetic energy at B.

c) Calculate its speed at point B.

d) Find the kinetic energy and potential energy at point C.

7.15 In problem 7.14, if the surface of the bowl is rough, and the speed at point B is 1.0 m/s.

a) Find the kinetic energy at B.

b) How much energy is lost as a result of friction as the particle goes from *A* to *B*?

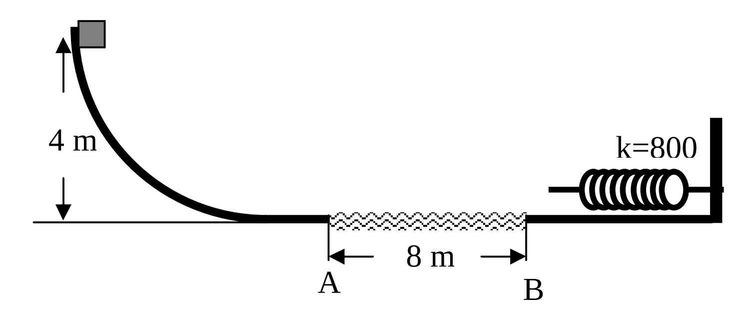


Figure 7.17 Problem 7.16.

7.16 A 8-kg block is released from rest at the top of the track shown in Figure 7.17. The track is 4-m high and smooth except for the portion *AB* whose length is 8 m, where μ_k = 0.4. At the end of the track the block hits a spring of force constant 800 N/m. What is the maximum compression of the spring?

7.17 A 10-kg block is connected to a light spring having a spring constant of 100 N/m on a rough inclined plane as shown in Figure 7.18. The block is released from rest when the spring is unstretched and the pulley is frictionless. The block moves 10 cm down the incline

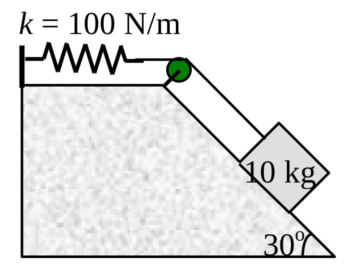


Figure 7.18 Problem 7.17.

before coming to rest. Find the coefficient of kinetic friction between the block and the incline.

7.18 A 3 kg block is pushed against a spring on a smooth 30^o incline as shown in Figure 7.19. The spring, whose force constant is 80 N/m, is compressed 10 cm and then released. How far along the incline the block will reach?

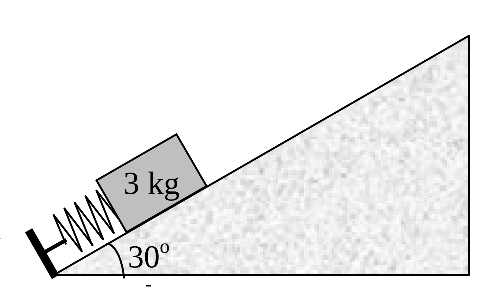


Figure 7.19 Problem 7.18.

7.19 A block of mass 8 kg is released from rest at the top of a smooth, 30^o inclined plane. At the bottom of the plane the block collides with a spring of force constant 1500 N/m. If the block starts at a height of 2.5 m from the equilibrium position of the spring, find the compressing distance of the spring before stops momentarily.

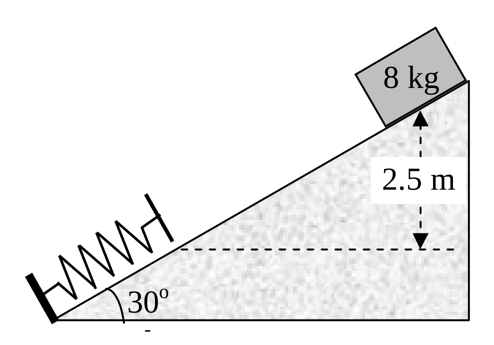


Figure 7.20 Problem 7.19.

7.20 Figure 7.21 shows two blocks of masses 10 kg and 20 kg which are connected by a rope that passes over a frictionless pulley. The 20-kg block is connected to unstreched spring of negligible mass and spring constant 200 N/m. The 10-kg block is pulled a

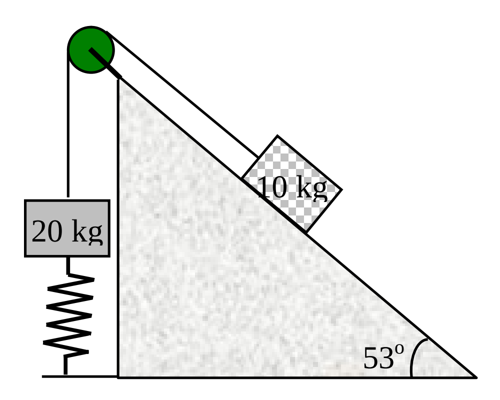


Figure 7.21 Problem 7.19.

distance of 20 cm down the smooth incline, and then released from rest. Find the speed of each block when the 20-kg block returns to its initial position.

7.21 A block is dragged down a 30°-incline a distance of 4 m by a 3-N force that is parallel to the plane, as shown in Figure 7.22. The frictional force that retard the motion is 6 N. If the kinetic energy of the block is increases by 32 J as it moves the 4-m distance,

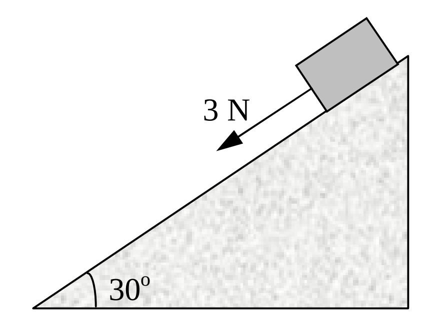


Figure 7.22 Problem 7.19.

a) What is the change in the potential energy as that block moves that distance?

b) What is the mass of the block?

7.22 Two masses m_1 = 40 kg and m_2 = 80 kg are connected by a string as shown in Figure 7.23. The pulley is frictionless and light. The coefficient of kinetic friction is μ_k=0.2. Find the change in kinetic energy of the system as m_1 moves a distance 20 m up the incline.

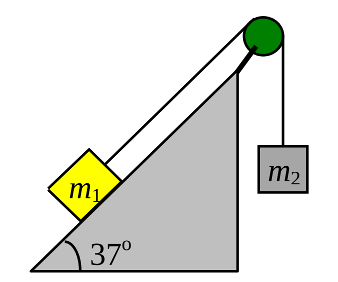


Figure 7.23 Problem 7.22.

7.23 In Figure 7.24, a mass *m* rests on a spring , compressing it a distance *d* from its equilibrium position. Then suppose that the mass is put on the unstretched spring with initial velocity equal to zero.

a) Find the maximum compression distance d_{max} (in terms of *d*) of the spring as the mass moves

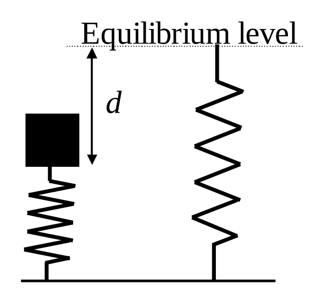


Figure 7.24 Problem 7.23.

downward.

b) Find the maximum speed of the mass.

7.24 A particle is placed on top of a smooth, hemispherical surface of radius R, as shown in Figure 7.25. The block is given a very small push and starts slides down the hemisphere. Show that the block will leave the surface at a point whose height is $\frac{2}{3}R$.

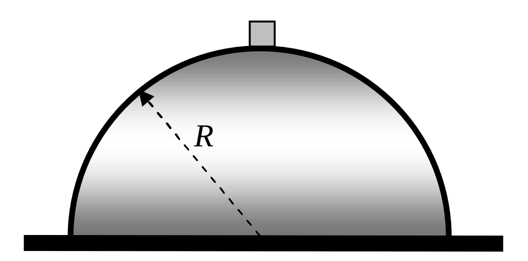


Figure 7.25 Problem 7.24.

CHAPTER 8

SYSTEMS OF PARTICLES

8.1 CENTER OF MASS

The center of mass of a system of particles or a rigid body is the point at which all of the mass are considered to be concentrated there and all external forces were applied there. In this section we want to know how to determine the center of mass of a system.

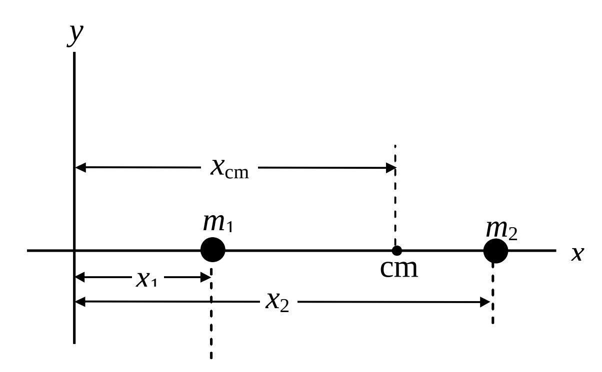


Figure 8.1 A system of two particles m_1 and m_2. The point labeled cm is the position of the center of mass of the system.

Consider a system of two masses m_1 and m_2 located along the x-axis as shown in Figure 8.1. The position x_{cm} of the center of mass of these two masses is defined to be

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}. \tag{8.1}$$

If $m_1 = m_2$, $x_1 = 0$, and $x_2 = d$, we find that $x_{cm} = \frac{d}{2}$, i.e., the center of mass lies midway between the two masses.

For a system of n-particles m_1, m_2, …, m_n, the center of mass is

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \mathbf{L}\, m_n x_n}{m_1 + m_2 + \mathbf{L}\, m_n} \tag{8.2}$$

$$x_{cm} = \frac{1}{M}\sum_{\mathrm{i}=1}^{\mathrm{n}} m_{\mathrm{i}} x_{\mathrm{i}} \qquad (8.3)$$

Where $M = m_1 + m_2 + \mathbf{L} + m_n$ is the total mass of the system.

If the system of particles is distributed on three dimension, the y and the z coordinates of the center of mass are similarly defined by

$$y_{cm} = \frac{1}{M}\sum_{\mathrm{i}=1}^{\mathrm{n}} m_{\mathrm{i}} y_{\mathrm{i}}\ , \qquad (8.4)$$

and

$$z_{cm} = \frac{1}{M}\sum_{\mathrm{i}=1}^{\mathrm{n}} m_{\mathrm{i}} z_{\mathrm{i}} \quad . \qquad (8.5)$$

In vector notation, the position vector of the center of mass $\mathbf{r}_{\mathrm{cm}}$ can be expressed as

$$\mathbf{r}_{\mathrm{cm}} = x_{\mathrm{cm}}\mathbf{i} + y_{\mathrm{cm}}\mathbf{j} + z_{\mathrm{cm}}\mathbf{k} \qquad (8.6)$$

Or

$$\mathbf{r}_{cm} = \frac{1}{M}\sum_{\mathrm{i}=1}^{\mathrm{n}} m_{\mathrm{i}}\mathbf{r}_{\mathrm{i}} \qquad (8.7)$$

To find the center mass of a rigid body (continuous mass distribution) we treat the body as consisting of so large number of small elements dm such that the sums of Equations 8.3-8.5 become integrals and the coordinates of the center of mass become

$$x_{\text{cm}} = \frac{1}{M}\int x dm \,, \tag{8.8}$$

$$y_{\text{cm}} = \frac{1}{M}\int y dm \,, \tag{8.9}$$

$$z_{\text{cm}} = \frac{1}{M}\int z dm \,, \tag{8.10}$$

The vector position of the center of mass of a rigid body is expressed as

$$\mathbf{r}_{\text{cm}} = \frac{1}{M}\int \mathbf{r} dm \,. \tag{8.11}$$

The integrals are to be evaluated over all the mass distribution of the object.

Example 8.1 Three particles of masses m_1 = 1 kg, m_2 = 2 kg, and m_3 = 3 kg are located as shown in Figure 8.10. Find the center of mass of this system.

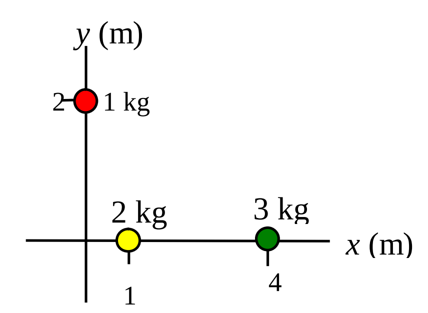


Figure 8.2 Example 8.1.

Solution The *x*-component of the center of mass is

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times 0 + 2\times 1 + 3\times 4}{1+2+3} = \frac{14}{6} = 2.33 \text{ m}.$$

The y-component is

$$y_{\text{cm}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$
$$= \frac{1\times 2 + 2\times 0 + 3\times 0}{1+2+3}$$
$$= \frac{2}{6} = 0.33\text{m}$$

The position of the center of mass is therefore

$$\mathbf{r}_{\text{cm}} = 2.33\mathbf{i} + 0.33\mathbf{j}\text{. m}$$

Example 8.2 Show that the center of mass of a uniform rod of mass M and length L lies midway between its ends.

Solution Let the rod be located along the x-axis as shown in Figure 8.3. By symmetry it is obvious that $y_{\text{cm}} = z_{\text{cm}} = 0$. Let us take a small element of mass dm and length dx. From Equation 8.8, we have

$$x_{\text{cm}} = \frac{1}{M}\int x\,dm$$

Figure 8.3 Example 8.2.

To solve the integral we need to find a relation between the mass dm and the variable x. To find such a relation we define the linear mass density λ (mass per unit length), as $l = \frac{dm}{dx} = \frac{M}{L}$. Now the above equation becomes

$$x_{\text{cm}} = \frac{l}{M}\int_0^L x dx = \frac{l}{M}\frac{x^2}{2}\Big]_0^L = \frac{lL^2}{2M}$$

Substituting for $l = \frac{M}{L}$, we get

$$x_{cm} = \left(\frac{M}{L}\right)\frac{L^2}{2M} = \frac{L}{2}.$$

8.2 DYNAMICS OF A SYSTEM OF PARTICLES

From Equation 8.7 we have

$$M\,\mathbf{r}_{cm} = \sum_{\mathrm{i}=1}^{\mathrm{n}} m_{\mathrm{i}}\mathbf{r}_{\mathrm{i}}$$

Differentiating the above equation with respect to time gives

$$M\frac{d(\mathbf{r}_{cm})}{dt} = \sum_{\mathrm{i}=1}^{\mathrm{n}} m_{\mathrm{i}}\frac{d(\mathbf{r}_{\mathrm{i}})}{dt} \tag{8.12}$$

Knowing that $\frac{d(\mathbf{r}_{cm})}{dt}$ is the velocity of the center of mass and $\frac{d(\mathbf{r}_{\mathrm{i}})}{dt}$ is the velocity of the i^{th} particle, Equation 8.12 becomes

$$M\mathbf{v}_{cm} = \sum_{\mathrm{i}=1}^{\mathrm{n}} m_{\mathrm{i}}\mathbf{v}_{\mathrm{i}} \tag{8.13}$$

Differentiating Equation 8.13 with respect to time leads to

$$M\mathbf{a}_{cm} = \sum_{i=1}^{n} m_i \mathbf{a}_i \qquad (8.14)$$

Where $\frac{d(\mathbf{v}_{cm})}{dt}$ is the acceleration of the center of mass and $\frac{d(\mathbf{v}_i)}{dt}$ is the acceleration of the i^{th} particle.

From Newton's second law we know that $m_i\mathbf{a}_i$ represents the resultant force $\mathbf{F}_i$ that acts on the i^{th} particle. Thus we can write Equation 8.14 as

$$M\mathbf{a}_{cm} = \sum_{i=1}^{n} \mathbf{F}_i \qquad (8.15)$$

Remember that $\mathbf{F}_i$ is the vector sum of the external forces acting on the i^{th} particle and the internal forces resulting from the other particles of the system. From Newton's third law the internal forces form action-reaction pairs so that they cancel out in the sum of Equation 8.15. So, the right hand side of Equation 8.15 is the vector sum of all the external forces $\mathbf{F}_{ext}$ that act on the system. Equation 8.15 then reduces to

$$M\mathbf{a}_{cm} = \sum \mathbf{F}_{ext} \qquad (8.16)$$

Equation 8.16, like any vector equation can written as three equations corresponding to the components of $\mathbf{a}_{cm}$ and $\mathbf{F}_{ext}$ along the coordinates axes, that is,

$$\sum F_x^{ext} = Ma_x^{cm}$$
$$\sum F_y^{ext} = Ma_y^{cm} \qquad (8.17)$$
$$\sum F_z^{ext} = Ma_z^{cm}$$

Equation 8.16 tells that if no net external force acting on a system, the acceleration of its center of mass is zero and thus the velocity of the center of mass of the system remains unchanged.

Example 8.3 A shell is fired with an initial speed v_0 at an angle of θ above the horizontal. At the top of the trajectory, the shell explodes into two equal fragments. One fragment, whose speed immediately after explosion is zero, falls vertically down, as shown in Figure 8.4. How far from the initial point does the other fragment land.

Solution

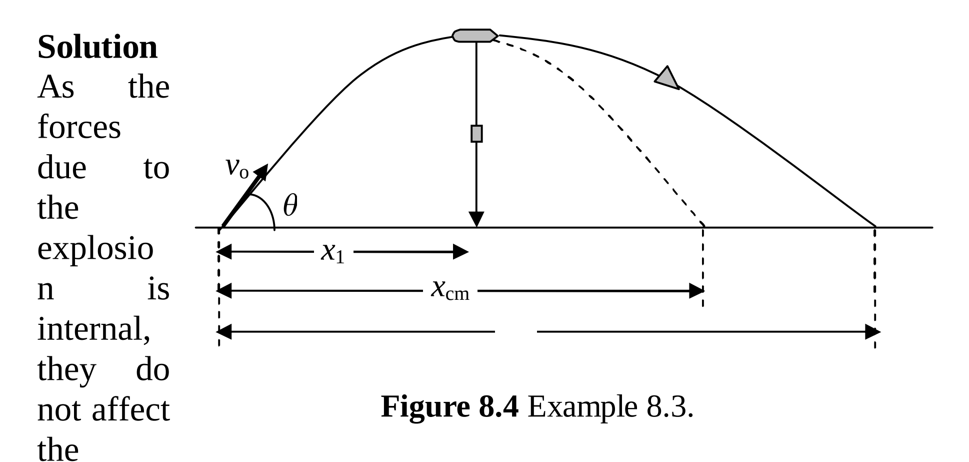


Figure 8.4 Example 8.3.

As the forces due to the explosion is internal, they do not affect the motion of the center of mass. Since the only external force acting on the system is the force of gravity, the center of mass follows a parabolic path (the dotted path shown in Figure 8.4) as the projectile did not explode. From Example 3.2 we obtain

$$x_{cm} = \frac{v_o^2 \sin 2q}{g}$$

But from Equation 8.3 we have

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{M}$$

Knowing that $x_1 = \frac{1}{2} x_{cm}$ and $m_1 = m_2 = \frac{1}{2} M$ we get

$$x_{\text{cm}} = \frac{\frac{1}{2} x_{cm} + x_2}{2}$$

or

$$x_2 = \frac{3}{2} x_{cm} = \frac{3}{2} \frac{v_o^2 \sin 2q}{g}$$

Example 8.4 A railroad car of mass *M* can move along a smooth horizontal track. A man of mass *m* is, initially standing at one end of the car, which is initially at rest, as shown in Figure 8.5. If the man starts to walk toward the other end of the car, describe the motion of the car.

Solution There is no external force acting on the man-car system along the horizontal direction. This means that the velocity of the center of mass of the system will not change and must remain zero, and so the position of the center of mass of the man-car system is the same before and after the man starts to walk.
To maintain the position of the center of mass unchanged, the car will move in a direction opposite to the direction of the walking man, as it clear from Figure 8.5.

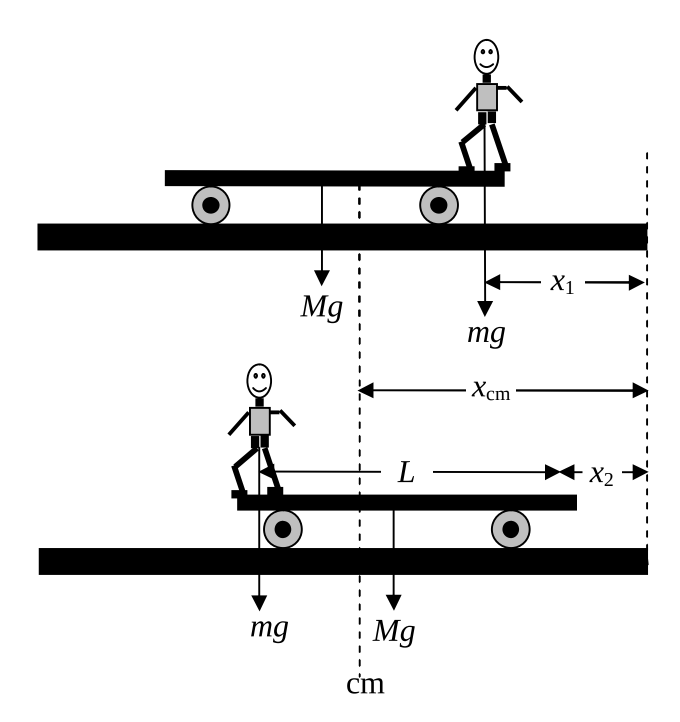


Figure 8.5 Example 8.4.

Let us now study the motion of the car analytically. Let x_1 be the position of right end of the car (at which the man is initially stands) relative to a fixed axis, and x_2 is the new position of the same end. Assuming that the car is uniform and the man walks a distance L between its ends, the position of the center of mass of the man-car system when the man is at the right end is

$$x_{cm} = \frac{mx_1 + M\left(x_1 + \frac{1}{2}L\right)}{m+M}$$

When the man is now at the left end of the car the position of the center of mass is

$$x_{cm} = \frac{m(x_2 + L) + M\left(x_2 + \frac{1}{2}L\right)}{m + M}$$

Equating the above two equations we obtain

$$x_2 = x_1 - \frac{m}{M + m}L$$

The last equation tells that if the man moves a distance L to the left, the car will move to the right a distance $mL/(M + m)$.

8.3 LINEAR MOMENTUM

The linear momentum **p** of a particle of mass *m* moving with velocity **v** is defined as

$$\mathbf{p} = m\mathbf{v} \qquad (8.18)$$

In SI unit system the momentum has a unit of kg.m/s. The components of momentum **p** in three dimensions are

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z.$$

Let us now find the relationship between the linear momentum and the force. From Newton's second law we have

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt}, \qquad (8.19)$$

where *m* is constant. Newton's second law thus can be written as

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad (8.20)$$

For a system of n particles, each with its own mass and velocity, the linear momentum **P** of the system as a whole is the vector sum of the linear momenta of each particle individually, that is

$$\begin{aligned} \mathbf{P} &= \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{L} + \mathbf{p}_n \\ &= m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \mathbf{L} + m_n\mathbf{v}_n = \sum_{\mathrm{i}=1}^{\mathrm{n}} m_\mathrm{i}\mathbf{v}_\mathrm{i} \end{aligned} \qquad (8.21)$$

Comparing this equation with Equation 8.13 we get

$$\mathbf{P} = M\mathbf{v}_{cm} \qquad (8.22)$$

From Equation 8.22 we define the linear momentum of a system of particles as the product of the total mass of the system and the velocity of its center of mass.
Differentiating Equation 8.22 with respect to time we find

$$\frac{d\mathbf{P}}{dt} = M\frac{d\mathbf{v}_{cm}}{dt} = M\mathbf{a}_{cm} \qquad (8.23)$$

Comparing Equations 8.16 and 8.23 we write

$$\sum \mathbf{F}_{\mathrm{ext}} = \frac{d\mathbf{P}}{dt} \qquad (8.24)$$

Which is the generalization of Newton's second law to a system of particles.

8.4 CONSERVATION OF LINEAR MOMENTUM

If a system is isolated, that is the resultant external force acting on the system is zero, Equation 8.24 gives

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} = 0,$$

or

$$\mathbf{P} = \text{constant.} \tag{8.25}$$

This important relation is called the conservation of linear momentum principle, which states that, if no net external force acts on a system, then the total momentum of the system remains constant. Equation 8.25 can be written as

$$\mathbf{P}_i = \mathbf{P}_f \tag{8.26}$$

This means that, for an isolated system, the linear momentum of the system at some initial point *i* is equal to the linear momentum at some final point *f*.
It is worth to mention that Equations 8.25 and 8.26 are vector equations and so both are equivalent to three separately equations corresponding the three perpendicular axes. Depending on the external force acting on the system, the linear momentum might be conserved in one or two directions, but not necessary in all directions. In another word, if only one component of the net external force acting on a system along an axis is zero, then the component of the linear momentum of the system along that axis is constant only. The other two components in this case is not constant.

Example 8.5 A cannon of mass 2000 kg rests on a smooth, horizontal surface as shown in Figure 8.6. The cannon fires, horizontally, a ball of mass 25 kg with a speed of 75 m/s relative to the earth. What is the velocity of the cannon just after it fires the ball?

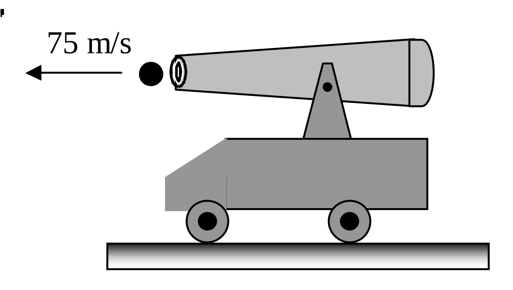


Figure 8.6 Example 8.5.

Solution We take our system to consists of the cannon and the cannonball. The two external forces, the force of gravity and the normal force, are both perpendicular to the motion of the system. Therefore, the *x*-component of the linear momentum of the system is conserved. Before firing the linear momentum of the system P_i is zero, while the linear momentum of the system just after firing P_f is

$$P_f = m_c v_c + m_b v_b$$

Where *c* and *b* refer, respectively, to the cannon and the cannonball. Conservation of linear momentum in the horizontal direction requires that

$$m_c v_c + m_b v_b = 0$$

Solving for v_c yields

$$v_c = -\frac{m_b}{m_c} v_b = -\left(\frac{25}{2000}\right) 75 = -0.94 \, \text{m/s}$$

The negative sign indicates that the cannon recoils to the right, in the direction opposite to the motion of the ball.

Example 8.6 Two blocks of masses m_1 = 1 kg and m_2 = 2 kg are connected by a spring of force constant k=200 N/m. The two blocks are free to slide along a frictionless horizontal surface, as shown in Figure 8.7. The blocks are pushed in opposite direction compressing the spring a distance of 12 cm, and then released from rest. Find the velocities of the two blocks when the spring returns to its equilibrium state.

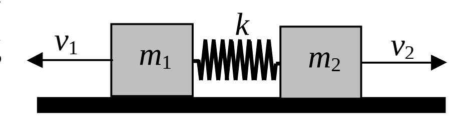


Figure 8.7 Example 8.6.

Solution Our system is the two blocks and the spring. Therefore, the total momentum in the horizontal direction is conserved. Knowing that the system is initially at rest we obtain

$$0 = m_1 v_1 + m_2 v_2$$

So we get

$$v_2 = -\frac{m_1}{m_2} v_1$$

Which gives the relation between the two velocities at any instant of the motion.
Now applying the conservation of mechanical energy principle we can write

$$\frac{1}{2} k x^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Substituting for v_2 from the previous equation we get

$$m_1 v_1^2 + m_2 \left(\frac{m_1}{m_2} \right)^2 v_1^2 = k x^2$$

Solving for v_1 we obtain

$$v_1 = \sqrt{\frac{km_2 x^2}{m_1 m_2 + m_1^2}} = \sqrt{\frac{(200)(2)(0.12)}{2+1}} = 4.0\,\text{m/s}$$

Again using the relation between the two velocities we get

$$v_2 = -\frac{m_1}{m_2} v_1 = -\frac{1}{2}(4.0) = -2.0\,\text{m/s}$$

PROBLEMS

8.1 Two particles are located along the x-axis. The first particle with mass m_1=4 kg has the coordinates (2m,0,0), while the other particle with mass m_2=8 kg has the coordinates (4m,0,0). Find the coordinates of the center of mass of the system.

8.2 A particle of mass 2 kg is located on x=-2 m, and a particle of mass 3 kg is on x=4 m. Find the position of the center of mass.

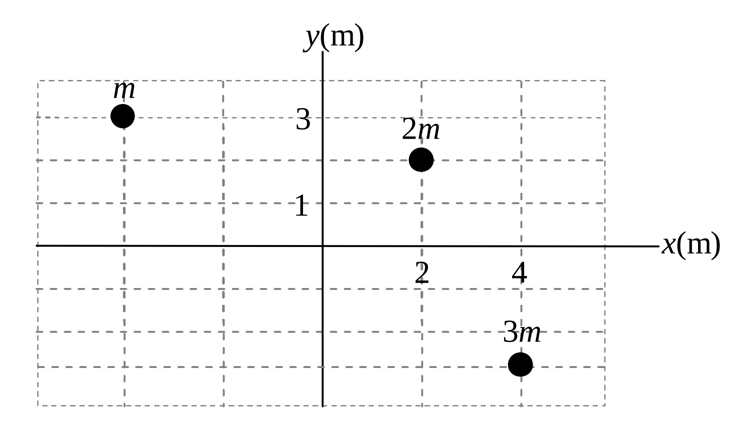


Figure 8.8 Problem 8.3.

8.3 Three masses are located as shown in Figure 8.8. Find the center of mass of the system.

8.4 Three thin rods each of length L is arranged as shown in Figure 8.9. The two vertical rod have equal mass M, and the horizontal rod has a mass 2M. Find the center of mass of the system.

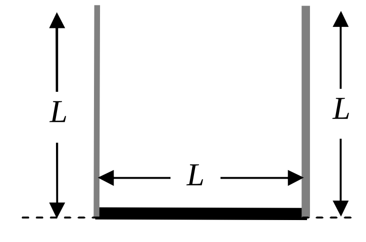


Figure 8.9 Problem 8.4.

8.5 Show that the center of mass for a rectangular plate of sides *a* and *b* is at the center of the plate.

8.6 A lamina in the shape of a right triangle, with dimensions *a* and *b* as shown in Figure 8.25, has a uniform mass per unit area. Find the coordinates of the center of mass of the lamina.

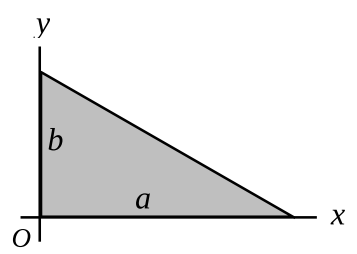


Figure 8.10 Problem 8.5.

8.7 A square piece of side 10 cm is cut out of a square plate of side 30 cm. Find the coordinates of the center of mass of the plate.

8.8 A 6 kg particle moves along the x-axis with a speed of 4 m/s.

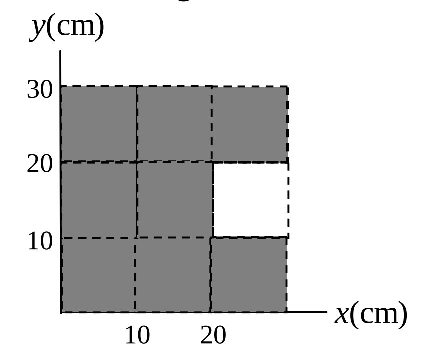


Figure 8.11 Problem 8.7.

A another particle of mass 4 kg moves along the x-axis with a speed of 8 m/s. Calculate the velocity of the center of mass.

8.9 Two boys one of mass 45 kg and the other of mass 35 kg, stand on a frictionless, horizontal surface holding a rigid rope of length 15 m. Starting from the ends of the rope, the boys pull themselves along the rope until they meet. How far will each boy move?

8.10 Two particles are initially at rest and 1.5 m apart. The two particles attract each other with a constant force of 2 mN,

which is the only force acting on the system. If the masses of the particles are 1 kg and 0.5 kg, find

a) the speed of the center of mass,

b) the distance from the first particle's position at which they collide.

8.11 At the instant the traffic light turns green, a car with mass 1500 kg starts from rest with constant acceleration of 2 m/s^2. At the same instant a truck of mass 3000 kg, traveling with constant speed of 10 m/s overtakes and passes the car.

a) How far beyond the traffic light is the center of mass of the car-truck system at t= 12 s?

b) What is the speed of the center of mass at that instant?

8.12 Consider a body of mass 2-kg and velocity of (2i-3j) m/s.

a) Find the x and y components of momentum.

b) The magnitude of its total momentum.

8.13 Calculate the magnitude of the linear momentum for the following cases:

a) a proton of mass 1.67×10^{-27} kg moving with a speed of 5×10^{6} m/s,

b) a 15-g bullet moving with a speed of 500 m/s,

c) a 75-kg man running at a speed of 10 m/s, and

d) the earth of mass 5.98×10^{26} kg, moving with an orbital speed of 2.98×10^{4} m/s.

8.14 Two blocks of masses m_1 = 3 kg and m_2 = 6 kg are connected by a spring of force constant k=800 N/m. The two blocks are free to slide along a frictionless horizontal surface, as shown in Figure 8.12. The blocks are pulled in opposite direction stretching the spring a distance of 10 cm, and then released from rest.

a) Find the velocities of the two blocks when the spring is at its equilibrium state.
b) What is the maximum compressing distance of the spring?

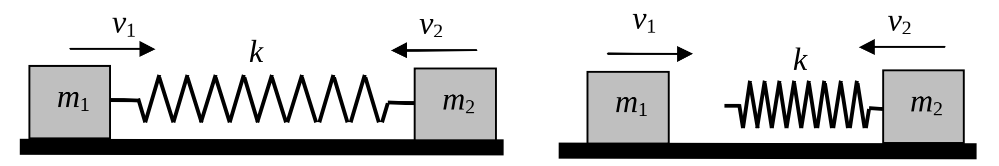


Figure 8.12 problem 8.14.   **Figure 8.13** problem 8.15.

8.15 A block of mass m_1 =2 kg moves with speed of v_1 =10 m/s to the right on a frictionless surface collides with a mass m_2=8 kg moving with velocity of v_2 =3 m/s to the right. A light spring of spring constant k=1000 N/m is connected to the block of mass m_2, as in Figure 8.13. When the blocks collide, what is the maximum compression of the spring?

8.16 A 60-kg student is standing on a cart of mass 120 kg. The cart, originally at rest, is free to slide on a smooth, horizontal surface. The student begins to walk along the cart at a constant velocity of 0.8 m/s relative to the cart.
a) What is the student's velocity relative to the ground?
b) What is the velocity of the cart relative to the ground?

8.17 A block of mass 6 kg sliding on a frictionless surface explodes into two equal pieces. One piece goes south at 4 m/s, and the other piece goes 30^o north of west at 5 m/s. What was the original velocity of the block?

8.18 A block of mass 10-kg initially at rest explodes into three pieces. A 4.5-kg piece goes north at 20 m/s, and a 2-kg piece moves eastward at 60 m/s.
a) Determine the magnitude and direction of the velocity of the third piece.

b) Find the energy of the explosion.

8.19 A cart of mass 40 kg is traveling at a speed of 3 m/s along a horizontal smooth surface. A 90-kg man initially riding on the cart jumps off with zero horizontal speed. What is the final speed of the cart?

8.20 A cannon car of mass 5000 kg that rests on a smooth, horizontal surface is attached to a post by a spring of force constant of 4×10^4 N/m, as shown in Figure 8.14. The cannon fires a ball of mass 150 kg with a speed of 100 m/s , directed 30^o above the horizontal.

a) What is the velocity of the cannon just after it fires the ball?

b) Find the maximum extension of the spring.

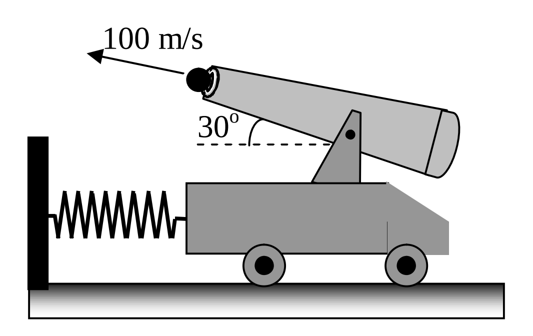


Figure 8.14 Example 8.20.

CHAPTER 9

ELECTRIC FIELD

9.1 ELECTRIC CHARGE

Two kinds of electric charges are found to exist which were given the names **positive** and **negative** by Benjamin Franklin (1706-1790). As it is known, atoms are composed of three elementary particles: the negatively charged **electrons**, the positively charged **protons** and **neutrons** with no net electric charge. The negative charge of an electron and the positive charge of a proton have the same magnitude. Thus, an atom is electrically neutral when the number of electrons equals the number of protons. It is found, experimentally, that like charges repel each other and unlike charges attract each other.

When two materials are brought together it is possible, under certain conditions, that electrons move from one material to the other. The material that gains electrons is said to be negatively charged, while the material that loses electrons is said to be positively charged. This explains the observation that when a rod of glass is rubbed with silk, the silk becomes negatively charged while the glass becomes positively charged. It is clear now from the above discussion that electric charge is not created, i.e., **electric charge is conserved**.

9.2 CONDUCTORS AND INSULATORS

According to their ability to conduct electric charge, substances are classified into two major classes: Conductors and insulators. A **conductor** is the substance that permits electric charge to move freely, whereas an **insulator** is the substance that does not permit electric charges to move. Silver, copper, and aluminum are examples of conductors while glass, plastic, and rubber are examples of insulators. There are substances that fall in an intermediate position between conductors and insulators. These substances are called **semiconductors** such as silicon and germanium.

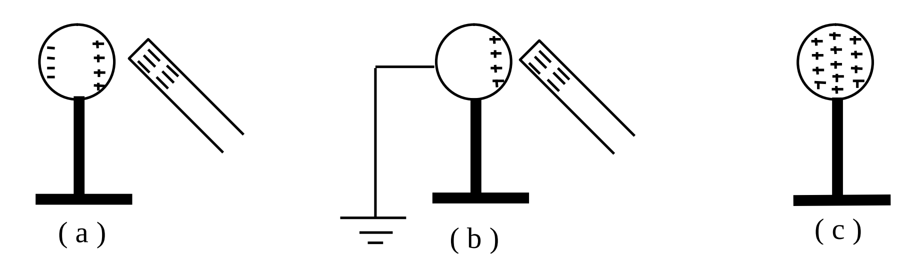


Figure 9.1 Charging by induction. (a) The charge on a conducting sphere is redistributed when a charged rubber rod is brought near the sphere. (b) The access negative charges leave the sphere as it is grounded. (c) The rubber and the ground connection are removed, and the sphere has a uniform positive charge.

With the properties of conductors and insulators in mind, one can charge a conductor by a technique called charging by **induction**, (see Figure 9.1).
(i) A negatively charged rubber rod is brought near a conducting sphere that is isolated from ground.
(ii) Some of the electrons on the sphere are repelled away from the rod leaving excess positive charges on the other part of the sphere. The electrons cannot escape from the sphere, why? Note that the sphere is still neutral.
(iii) When the sphere connected to the ground (grounded), the electrons flow to the ground.
(iv) After disconnecting the sphere from the ground, the rod is now removed and the excess positive charge distributed uniformly on the surface of the sphere due the repulsive force between the positive charges.

9.3 COULOMB'S LAW

Using particle with small size compared to the distance between them (**point charges**), It is found the following: (i) The force is directly proportional to the magnitude of the charges of each particle. (ii) The force is inversely proportional to the square of the distance between the two particles. (iii) The direction of the force is along the line joining the two particles. (iv) The force is attractive if the charges are of opposite signs and repulsive if the charges have the

same signs. From these observations the magnitude of the force acting on a point charge q_1 due to another point charge q_2 can be expressed as

$$F_{12}=k\frac{|q_1||q_2|}{r_{12}^2} \quad 9.1$$

where r_{12} is the distance between the two charges, and k is the proportionality constant known as **Coulomb's constant**. The direction of the force is determined according to the sign of the two charges. It is repulsion if the two charges are alike and attractive if they are unlike.

The proportionality constant, k, has a numerical value depends on the choice of units. The SI unit of charge is Coulomb (C), which is considered as the fourth primary unit beside meter, kilogram, and second. In this system k has the value

$$k \cong 9.0\times10^9 \text{ N.m}^2/\text{C}^2$$

Coulomb's constant is usually written as

$$k=\frac{1}{4pe_{o}} \quad 9.2$$

with $e_o = 8.8542 \times 10^{-12}$ $C^2/N.m^2$, is called the **permittivity of free space**. The cgs unit of charge is statcoulomb or electrostatic unit (esu). In this system, the constant k is defined to be unity with no units.

As mentioned in Section 9.1, the magnitude of charge of an electron or a proton is the smallest unit of charge. This quantity, denoted by e, has the approximate value

$$e \cong 1.6 \times 10^{-19}\,\text{C}$$

Electric charge is **quantized**. That is, it must be found as integral multiple of e, i.e., $q = Ne$, where N is some integer.

In vector form Equation 9.1 can be written as

$$\boldsymbol{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{\boldsymbol{r}}_{12} \qquad 9.3$$

where $\hat{\boldsymbol{r}}_{12}$ is a unit vector directed from q_2 to q_1. It should be noted that the force given by Equation 9.3 is of the same form as the gravitational force. As the gravitational force is conservative, we conclude that the electrostatic force is also conservative.

If there are more than two charges, the total force acting on one charge due to the others is the vector

sum of the forces due to the individual charges. For example, if there are three charges, then the net force on particle 1 due to the other two particles is given as

$$\boldsymbol{F}_1=\boldsymbol{F}_{12}+\boldsymbol{F}_{13} \qquad 9.4$$

Always remember that the electric force obeys Newton's third law, that is

$$\boldsymbol{F}_{12}=-\boldsymbol{F}_{21} \qquad 9.5$$

Remark: Coulomb's law applies only for point charges.

Example 9.1 Two-point charges lie along the x-axis as shown in Figure 9.2. $q_1=6.0m\mathrm{C}$ is at the origin, and $q_2=4.0m\mathrm{C}$ is at x = 3.0 m. Find the electric force acting on charge q_1.

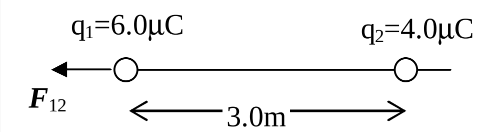


Figure 9.2 Example 9.1

Solution Using Equation 9.1 we write

$$F_{12} = k\frac{|q_1||q_2|}{r_{12}^2}$$

$$= \frac{(9\times10^9)(6\times10^{-6})(4\times10^{-6})}{(3)^2} = 2.4\times10^{-2}\,\text{N}$$

The direction of the force as shown in Figure 9.2 is along the negative x-axis because the two charges are positive. The force on q_1 is now written completely as

$$\boldsymbol{F}_{12} = -0.024\boldsymbol{i}\ \text{N}.$$

Example 9.2 Three point charges of 5.0 mC, -4.0 mC, and 8.0 mC are located at the corners of an equilateral triangle of side 2.0 m as shown in Figure 9.3. Find the resultant electric force exerted on q_1, the charge at the left corner of the triangle.

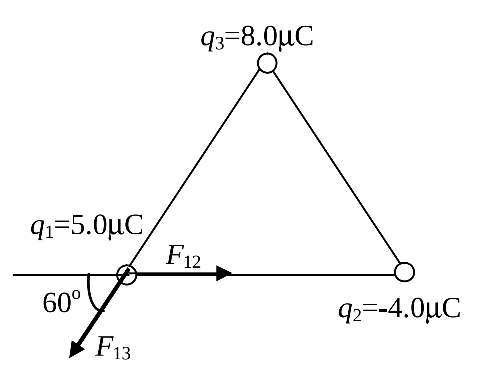


Figure 9.3 Example 9.2

Solution Here the charge q_1 are affected by two forces, F_{12} due to q_2 and F_{13} due to q_3. Now we use Equation 9.1 to obtain

$$F_{12} = k\frac{|q_1||q_2|}{r_{12}^2} = \frac{(9\times10^9)(5\times10^{-6})(4\times10^{-6})}{(2)^2} = 4.5\times10^{-2}\,\text{N}$$

and

$$F_{13} = k\frac{|q_1||q_3|}{r_{13}^2} = \frac{(9\times10^9)(5\times10^{-6})(8\times10^{-6})}{(2)^2} = 9.0\times10^{-2}\,\text{N}$$

As shown in the figure, the direction of $\boldsymbol{F}_{12}$ is along the positive x-axis while $\boldsymbol{F}_{13}$ is directed 60$^{\text{o}}$ south of east. This means that we can write the two forces in a vector form as

$$\boldsymbol{F}_{12} = 0.045\boldsymbol{i}\,\text{N}$$

and

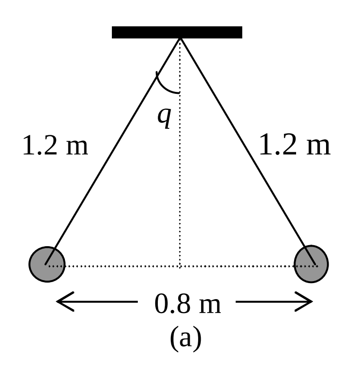
$$\boldsymbol{F}_{13} = -0.09\cos 60^{\text{o}}\,\boldsymbol{i} - 0.09\sin 60^{\text{o}}\,\boldsymbol{j}$$

$$= (-0.045\boldsymbol{i} - 0.078\boldsymbol{j})\text{N}$$

Now the resultant force acting on q_1 is the sum of the two forces, i.e.,

$$\boldsymbol{F}_1 = \boldsymbol{F}_{12} + \boldsymbol{F}_{13} = -7.8\times10^{-2}\,\boldsymbol{j}\,\text{N}$$

Example 9.3 Two identical small charged spheres, each of mass 0.100 kg and charge *q* are hanged in equilibrium as shown in Figure 9.4a. The length of each string is 1.20 m, and the separation distance between the two spheres is 80.0 cm. Find the magnitude of the electric charge on each sphere.



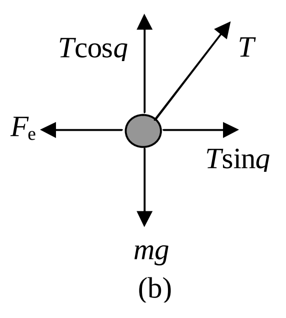


Figure 1.4 Example 1.3.

Solution It is enough to study the equilibrium state of one of the spheres. The forces acting on such a sphere is shown in the free-body diagram shown in Figure 9.4b. The equilibrium conditions imply that

$$\sum F_x = 0$$

and

$$\sum F_y = 0$$

From the first condition we write

$$F_e = T \sin q \qquad (1)$$

Here F_e is the electric force acting on the sphere by the other. The second condition gives

$$mg = T\cos q \qquad (2)$$

Dividing Equation (1) by Equation (2), we obtain

$$\tan q = \frac{F_e}{mg} \qquad (3)$$

It is clear from the triangle of Figure 9.4a that

$$\sin q = \frac{0.4}{1.2}\text{ , or } q = 19.5^{o}$$

Substituting the values of *m*, *g*, and *q* in Equation (3) we get

$$F_e = 0.35\,\text{N}$$

The magnitude of the electric force F_e is obtained from Coulomb’s law as

$$F_e = k\,\frac{q^2}{r^2}$$

Solving for q we get

$$q = \sqrt{\frac{F_e r^2}{k}} = \sqrt{\frac{0.35(0.8)^2}{9.0\times10^9}} = 5.00 m\text{C}$$

9.4 THE ELECTRIC FIELD

In analogy to the concept of gravitational field, any charge has its own electric field within a region surrounding the charge. The electric field ***E*** at a point in space is a vector quantity defined as the electric force acting on a positive test charge q_o placed at that point divided by the magnitude of the test charge, i.e.,

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{q_o} \qquad 9.6$$

The test charge should be small enough to ensure that its presence does not affect the charge distribution that produces ***E***. Since q_o is always positive, the direction of ***E*** is the direction of ***F***. The SI unit of ***E*** is Newton per Coulomb (N/C). If a charge q is placed at a point with an electric field **E**, this charge experiences an electric force given by

$$\boldsymbol{F} = q\boldsymbol{E} \qquad 9.7$$

Example 9.4 Find the electric force on an electron placed in an electric field of 1.5×10^{3} N/C directed along the positive *x*-axis.

Solution The force acting on the electron is obtained from Equation 9.7, that is,

$$\boldsymbol{F} = q\boldsymbol{E}$$

But for the electron $q = e = -1.6\times10^{-19}\ \mathrm{C}$, so we obtain

$$\boldsymbol{F} = (-1.6\times10^{-19})(1.5\times10^{3})\boldsymbol{i} = 2.4\times10^{-16}\,\boldsymbol{i}\,\mathrm{N}$$

Example 9.5 An electron is projected, horizontally with initial speed of 5×10^{8} m/s into a region of uniform electric field of 2×10^{6} N/C and directed vertically upward. After traveling a horizontal distance of 5 cm, find

a) the time required for the electron to travel this distance,

b) the vertical displacement of the electron.

Solution Let us first find the acceleration of the electron. Using Newton's second law we write

$$\boldsymbol{a}=\frac{\boldsymbol{F}}{m}=\frac{q\boldsymbol{E}}{m}$$

$$=\frac{\left(-1.6\times10^{-19}\right)2\times10^{6}\,\boldsymbol{j}}{9.11\times10^{-31}}=-3.51\times10^{17}\,\boldsymbol{j}$$

where we have substituted for ***F*** from Equation 9.7. Since ***E*** is uniform then ***F*** and so ***a*** are constants.

a) From the last result we conclude that $a_x = 0$. This means that we can write

$$t=\frac{x}{v_x}=\frac{0.05}{5\times10^{8}}=1\times10^{-10}\,\text{s.}$$

b) The vertical displacement can be calculated from the equation

$$y=v_{0y}t+\tfrac{1}{2}a_y t^2$$

$$=\tfrac{1}{2}\left(3.51\times10^{17}\right)\left(1\times10^{-10}\right)^2=-0.18\,\text{cm.}$$

9.5 ELECTRIC FIELD OF POINT CHARGES

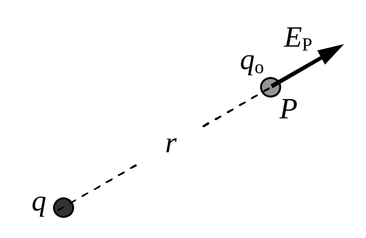


Figure 1.5 The electric field at point *P* a distance r from a point charge *q*.

Let us calculate the electric field arising from an isolated point charge q at a point p, a distance r from it. To do so we assume the existence of a test charge q_o at point p, (see Figure 9.5). From Coulomb's law (Equation 9.3) the force upon the test charge is

$$\boldsymbol{F} = k\frac{q\ q_o}{r^2}\hat{\boldsymbol{r}}$$

where the unit vector directed from q to the point p. Using Equation 9.6 we conclude that the electric field at point p is given as

$$\boldsymbol{E} = k\frac{q}{r^2}\hat{\boldsymbol{r}} \qquad 9.8$$

It is clear from the definition of $\hat{\boldsymbol{r}}$ that if q is positive the direction of $\boldsymbol{E}$ is outward from the charge, and if the charge is negative $\boldsymbol{E}$ is directed toward q.

The electric field at a point due to a group of point charges is the vector sum of the electric fields at that point due to each charge individually, i.e., if we have n charges the net electric field $\boldsymbol{E}$ is

$$\boldsymbol{E}=\boldsymbol{E}_1+\boldsymbol{E}_2+\cdots+\boldsymbol{E}_n \qquad 9.10$$

where $\boldsymbol{E}_1$, $\boldsymbol{E}_2$, and $\boldsymbol{E}_n$ are the electric fields due to the charges q_1, q_2, and q_n, respectively.

Finally, a spherical shell of uniform charge can be treated as if all its charge are concentrated at its center as far as we are concerned with the region outside the shell. This means that the electric field at a point outside a spherical shell with radius R and uniform charge Q is

$$\boldsymbol{E}=k\frac{Q}{r^2}\hat{\boldsymbol{r}} \qquad r \geq R \qquad (9.11)$$

The same approach holds for uniformly charged solid sphere since such a sphere can be considered as many concentric spherical shells.

Example 9.6 Two point charges of 24 μC and 8.0 μC are placed on the x-axis 0.3 m apart as shown in Figure 9.6.

a) Find the electric field at the midpoint between the two charges

b) Calculate the electric force acting on a third charge of -4 μC placed at the midpoint.

q_1=24μC E_2 E_1 q_2=8.0μC

0.3 m

Figure 1.6 Example 1.6

Solution a) Let us first calculate the magnitudes of the electric fields for each charge individually, E_1 due to the 24-μC charge and E_2 due to the 8-μC.

$$E_1 = k\frac{|q_1|}{r_1^2} = (9.0\times10^9)\frac{24\times10^{-6}}{(0.15)^2} = 9.6\times10^6 \text{ N/C}$$

and

$$E_2 = k\frac{|q_2|}{r_2^2} = (9.0\times10^9)\frac{8\times10^{-6}}{(0.15)^2} = 3.2\times10^6 \text{ N/C}$$

Since the two charges are positives, $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are directed outward from the respective charge as shown in Figure 9.6. Hence, we can write

$$\boldsymbol{E}_1 = 9.6\times10^6 \boldsymbol{i} \text{ N/C}$$

$$\boldsymbol{E}_2 = -3.2\times10^6 \boldsymbol{i} \text{ N/C}$$

The net electric field at the midpoint is now

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 = 6.4 \times 10^6 \, \boldsymbol{i} \text{ N/C}$$

b) The electric force on the third charge can be calculated directly as

$$\boldsymbol{F} = q_3 \boldsymbol{E}$$

where $\boldsymbol{E}$ is the electric filed at the point where q_3 is placed. So we found

$$\boldsymbol{F} = \left(-4 \times 10^{-6}\right)\left(6.4 \times 10^{6}\right)\boldsymbol{i} = 26 \text{ N}$$

9.6 THE ELECTRIC FIELD LINES

The electric filed in a region is represented by imaginary lines known as **electric field lines** (lines of force) introduced by Michael Faraday (1791-1867). These lines have the following properties:

1- The direction of the lines at any point is the direction of the electric field at that point.

2- The lines must begin on positive charges and terminate at negative charges.

3- The number of lines per unit area, perpendicular to $\boldsymbol{E}$, is proportional to the magnitude of $\boldsymbol{E}$ in that region.

4- No two field lines can cross.
Figure 9.- shows the electric field lines drawn for some common charge configurations.

9.7 ELECTRIC DIPOLE

A configuration of two charges of magnitude *q* but of opposite sign, separated by a distance *d* is called **electric dipole**. Let us calculate the electric field due to the electric dipole shown in Figure 9.8 at point P along the *y*-axis a distance *y* from the origin. The resultant electric field at P is $\boldsymbol{E}=\boldsymbol{E}_{+}+\boldsymbol{E}_{-}$, where $\boldsymbol{E}_{+}$ and $\boldsymbol{E}_{-}$ are the electric field at P due to the positive and the negative charge respectively. Since P is equidistant from the two charges the magnitude of $\boldsymbol{E}_{+}$ and $\boldsymbol{E}_{-}$ are equal and are given by

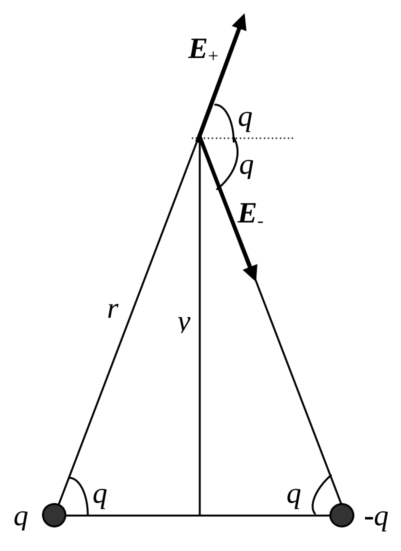


Figure 1.8 The electric field due to an electric dipole at a point *P*.

$$E_{+}=E_{-}=k\frac{q}{r^{2}}$$

As it is clear from the directions of the two fields shown in Figure 9.6, the y-components of $\boldsymbol{E}_+$ and $\boldsymbol{E}_-$ cancel each other. The net electric field at P is, therefore

$$\boldsymbol{E} = \left(E_+ \cos q + E_- \cos q\right)\boldsymbol{i}$$

From the figure we see that $\cos q = d/2r$, so we find

$$\boldsymbol{E} = 2k\frac{q}{r^2}\cos q = k\frac{qd}{r^3}\boldsymbol{i} \qquad 1.11$$

Now

$$r^3 = \left(\frac{d^2}{4} + y^2\right)^{3/2}$$

If $y \rangle\rangle d$ we can neglect the first term in the bracket so that r^3 is approximated to $r^3 \approx y^3$. Equation 9.10 can be written as

$$\boldsymbol{E} = k\frac{qd}{y^3}\boldsymbol{i} = k\frac{\boldsymbol{p}}{y^3} \qquad 9.12$$

where $\boldsymbol{p} = qd\boldsymbol{i}$ is called the **electric dipole moment**. The direction of $\boldsymbol{p}$ is taken to be from the negative charge to the positive charge.

Consider an electric dipole exists in an external uniform electric filed $\boldsymbol{E}$ that makes an angle q with the dipole moment $\boldsymbol{p}$. A force $\boldsymbol{F}_{+} = q\boldsymbol{E}$ in the direction of $\boldsymbol{E}$ is exerted on the positive charge; and a force $\boldsymbol{F}_{-} = q\boldsymbol{E}$ in a direction opposite to $\boldsymbol{E}$ is exerted on the negative charge. The net force acting on the dipole is zero. However, since the two forces do not have the same line of action, these two forces exert a torque t on the dipole about its center of mass. This torque is expressed as

$$t = F_{+}\frac{d}{2}\sin q + F_{-}\frac{d}{2}\sin q = qEd\sin q \qquad 9.13$$

Noting that $qd = p$, Equation 9.12 can be written as

$$t = pE\sin q \qquad 9.14$$

Equation 9.13 can be generalized to be written as

$$t = \boldsymbol{p} \times \boldsymbol{E} \qquad 9.15$$

This means that this torque tends to rotate the dipole into the direction of $\boldsymbol{E}$.

9.8 CONDUCTORS AT ELECTROSTATIC QUILIBRIUM

As mentioned in section 9.2 a conductor is the substance that permits electric charge to move freely. Charged Conductors with no motion of charges are said to be in electrostatic equilibrium. Such conductors have the following property:

1) The electric field inside a conductor is zero.

If it is not the case, the free electrons inside the conductor will be affected by an electric force ($\mathbf{F}=q\mathbf{E}$) and so will move which violate the equilibrium condition**.**

2) Any excess charge will reside entirely on the outer surface of an isolated conductor.

If a conductor is charged, charges will move a way from each other due to the repulsion force between them. For the charges to be as far away from each other as they can, they will move to the outer surface of the conductor.

3) The electric field just outside a conductor is always perpendicular to the surface of conductors.

If this is not the case, the free charges will move along the surface and again this violates the condition of equilibrium.

For special case the electric field just outside a spherical conductor of radius R can be calculated using Equation 9.11 as

$$\boldsymbol{E}=k\,\frac{Q}{R^2}\,\hat{\boldsymbol{r}}=\frac{Q}{4\pi\varepsilon_o R^2}\,\hat{\boldsymbol{r}}$$

Which directed as it clear radially. Now defining the surface charge density σ as charge per unit area we have $\sigma=\frac{Q}{A}=\frac{Q}{4\pi R^2}$. From which the electric field on the surface of the conducting sphere can be expressed as

$$E=\frac{\sigma}{\varepsilon_o} \qquad (9.16)$$

Even though Equation 9.16 is derived for spherical conductor, it is valid for any conductor. That is the magnitude of the electric filed on the surface of a conductor is given by Equation 9.16

CHAPTER 10

ELECTRIC POTENTIAL AND CAPACITANCE

10.1ELECTRIC POTENTIAL ENERGY

Consider a charged particle of charge q in a region of an electric field $\boldsymbol{E}$. This filed exerts an electric force on the particle given by $\boldsymbol{F}=q\boldsymbol{E}$. When the particle moves from initial point i to final point f this force does work on it according to

$$W=\int_i^f q\boldsymbol{E}\cdot d\boldsymbol{l} \qquad 10.1$$

where d$\boldsymbol{l}$ is an infinitesimal displacement along an arbitrary path from point i to point f. In section 9.3 we mentioned that the electrostatic force is conservative and so we can associate a potential energy with this force. The work done by a conservative force is defined to be equal to the negative of the change in the potential energy, that is

$$\Delta U=-W \qquad 10.2$$

or

$$U_f-U_i=-q\int_i^f \boldsymbol{E}\cdot d\boldsymbol{l} \qquad 10.3$$

Let us now define the electric potential V as the electric potential energy per unit charge, i.e.,

$$V=\frac{U}{q} \qquad 10.4$$

The electric potential difference between the points i and f is then follows from Equation 10.3 as

$$V_f - V_i = -\int_i^f \boldsymbol{E} \cdot d\boldsymbol{l} \qquad 10.5$$

Equation 10.5 admits the following definition:

> *The potential difference V_f - V_i is defined as the work required to move a positive unit charge from point i to point f.*

Since it is only the change in potential between two points that has physical sense, it is often convenient to choose a reference point for zero potential. This reference point is usually chosen to be at infinity, i.e., $V_\infty = 0$. Letting the point i to be ∞ and designate point f as point p Equation 10.5 becomes

$$V_p = -\int_\infty^p \boldsymbol{E} \cdot d\boldsymbol{l} \qquad 10.6$$

This gives the electric potential at any point p which can be defined as the work required to bring a positive unit charge from infinity to that point. In this sense the potential at a point is the potential difference between that point and a point at infinity.

As it is clear from the definition given by Equation 10.4, the electric potential is a scalar quantity. Therefore, dealing with electric potential is easier than dealing with electric field which is

vector. Also Equation 10.4 tells that the SI unit of V is joules per coulomb (J/C), usually represented by a special unit called **volt** (V) after Alessandro Volta(1745-1827), i.e.,1 V = 1 J/C

It follows from Equation 10.5 that the electric field has a unit of volt per meter (V/m) with1 N/C = 1 V/m

Example 10.1 A uniform electric field E is directed along the x-axis as shown in Figure 10.1. Calculate the potential difference between two points separated by a distance d, where d is measured a long the direction of $\boldsymbol{E}$.

Solution The two points are labeled A and B in the figure. Applying Equation 10.5 we get

Figure 10.1 Example 10.1.

$$\Delta V = V_B - V_A = -\int_A^B \boldsymbol{E} \cdot d\boldsymbol{l} = -\int_A^B E dl$$

This follows from the fact that $\boldsymbol{E}$ and d$\boldsymbol{l}$ are parallel. Since E is uniform it can be taken out from the integral sign, giving

$$\Delta V = -E \int_A^B dl = -Ed$$

From the minus sign we conclude that $V_A > V_B$.

Example 10.2 A proton is released from rest in a uniform electric field of 2.0×10^5 N/c directed along the positive x-axis, as shown in Figure 10.2. Find the speed of the proton after it has been displaced by 25 cm.

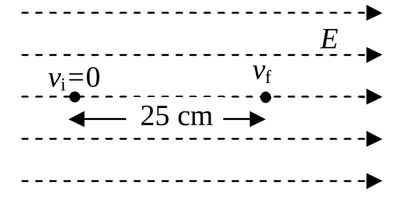


Figure 10.2 Example 10.2.

Solution Since the only force acting on the proton is the electrostatic force, we can apply the principle of conservation of mechanical energy in the form

$\Delta K + \Delta U = 0$, with $\Delta U = q\Delta V$.

or

$$\left(\tfrac{1}{2} m_p v^2 - 0\right) + e\Delta V = 0$$

Solving for v we get

$$v = \sqrt{\frac{-2e\Delta V}{m_p}}$$

To find ΔV we use

$$\Delta V = V_B - V_A = -\int_i^f \boldsymbol{E} \cdot d\boldsymbol{l} = -\int_i^f E dl$$

Note that the displacement of the positive proton is in the direction of **E**. So we have

$$\Delta V = -E\int_i^f dl = -2.0\times 10^5 (0.25) = -5.0\times 10^4 \text{ V}$$

And for the speed we obtain

$$\sqrt{\frac{2\left(1.6\times 10^{-19}\right)\left(5.0\times 10^4\right)}{1.67\times 10^{-27}}} = 3.1\times 10^6 \text{ m/s}$$

10.2ELECTRIC POTENTIAL DUE TO POINT CHARGES

Consider an isolated point charge q. To find the electric potential due to this charge at a point, we apply Equation 10.5 along an arbitrary path as shown in Figure 10.3. If $d\boldsymbol{l}$ is an element along the path and a distance r from q, the potential difference between any two points A and B along the path is

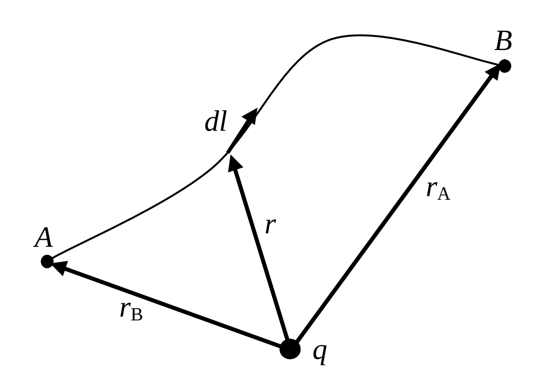


Figure 10.3 The electric potential at appoint a distance r from a point charge.

$$V_B - V_A = -\int_A^B \boldsymbol{E} \cdot d\boldsymbol{l} \qquad 10.7$$

The electric field due to the charge q at a distance r is

$$\boldsymbol{E} = k\frac{q}{r^2}\hat{\boldsymbol{r}},$$

where $\hat{\boldsymbol{r}}$ is a unit vector directed radially outward from q. Therefore, Equation 10.7 becomes

$$V_B - V_A = -kq\int_a^b \frac{\hat{\boldsymbol{r}} \cdot d\boldsymbol{l}}{r^2} \qquad 10.8$$

But $\hat{\boldsymbol{r}} \cdot d\boldsymbol{l}$ gives the projection of $d\boldsymbol{l}$ along the radial direction, i.e.,

$$\hat{\boldsymbol{r}} \cdot d\boldsymbol{l} = dr$$

Substituting back into Equation 10.7 and integrating we get

$$V_B - V_A = kq\left[\frac{1}{r}\right]_a^b = k\frac{q}{b} - k\frac{q}{a} \qquad 10.9$$

The first term of the right hand side of Equation 10.8 represents the potential at point B (V_B) and the second term represents the potential at point A (V_A). The electric potential at a point a distance r from a point charge q is then obtained if $a \to \infty$ in Equation 10.9, that is,

$$V = k\frac{q}{r} \qquad 10.10$$

As it is clear from the above equation, V is positive for positive q and negative for negative q.

The electric potential due to a group of point charges at a point is the algebraic sum of the electric potentials due to each charge individually. That is, for a group of N point charges we have

$$V = k\sum_{i}^{N}\frac{q_i}{r_i} \qquad 10.11$$

where r_i is the distance from the *ith* charge q_i to the point in question. Do not forget that the sum in Equation 10.11 is algebraic sum and not vector sum like that used to calculate the electric filed due to a group of point charges. This fact gives an important advantage of potential over electric field.

To find the electric potential at a point outside a spherical shell we again treat such a shell as if all its charges are concentrates at its center. This implies that such a potential will be given by Equation 10.10 but now r represents the distance between the point and the center of the shell.

Example 10.3 Three point charges of q_1= -4.0 μC, q_2= 12 μC, and q_3=16 μC are arranged as shown in Figure 10.4. Find the electric potential at the point *P*.

Solution From Equation 10.11 we write for N=3

$$V = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

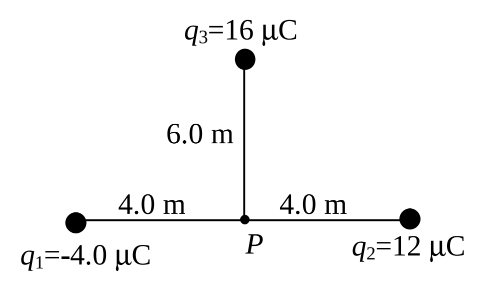


Figure 10.4 Example 10.3.

As it is clear from the figure, the distances from each charge to the point p are r_1=4.0 m, r_2=4.0 m, and r_3=6.0 m. Substituting for these values and the values of the charges in the above formula we get

$$V = 9\times10^{9}\left(\frac{-4.0}{4.0} + \frac{12}{4.0} + \frac{16}{6.0}\right)\times10^{-6}$$
$$= 4.2\times10^{4}\text{V}$$

Note that the negative sign of q_1 is included in our calculation.

10.3 POTENTIAL ENERGY OF A SYSTEM OF POINT CHARGES

We now want to calculate the electrostatic energy associated with a system of point charges. By this we mean the work required to assemble this system from a condition where all charges are infinitely separated from one another. To do so we have to calculate the work required to bring, from infinity, each charge one by one.

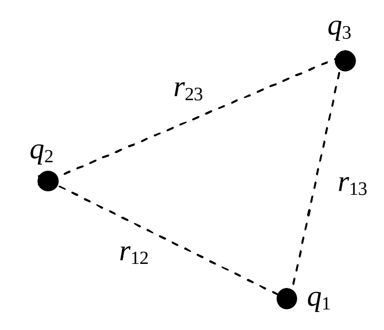


Figure 10.5 The potential energy of a system of three-point charges.

As defined in section 10.1, the potential at a point is equal to the work required to bring a positive unit charge from infinity to that point. Therefore, the work required to bring a charge q from infinity to a point must equal to the potential at that point multiplied by q, i.e.,

$$W_{\infty \to p} = qV_p \qquad 10.12$$

The minus sign of Equation 10.2 is dropped here because we are speaking with the work done by an external agent rather that the work done by the field.

Suppose that we want to assemble a system of three point charges as shown in Figure 10.5. According to Equation 10.12, no

work is required to place the first charge q_1 at a given position because there is no electric potential at that position. Next we place a second charge q_2 at a position r_{12} from q_1. This requires a work q_2V_1 where V_1 is the potential at the location of q_2 due to q_1. If we denote this work by W_2 with W_1=0 denotes the work required to place q_1, then we have

$$W_2 = k\frac{q_1q_2}{r_{12}}$$

Next we place q_3 at a position r_{13} from q_1 and r_{32} from q_2. We now must do work given as q_3V_{12}, where V_{12} is the potential at the location of q_3 due both q_1 and q_2, i.e.,

$$W_3 = kq_3\left(\frac{q_1}{r_{31}} + \frac{q_2}{r_{32}}\right)$$

The total work required to assemble a system of three charges is then

$$W = W_1 + W_2 + W_3$$

This work is stored as an electrostatic energy in the system, so we write

$$U = W = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) \qquad 10.13$$

The potential energy of a system of N charges can be calculated in a similar fashion. The result can be written as

$$U = k\sum_{j=1}^{N}\sum_{k=1}^{j-1}\frac{q_j q_k}{r_{jk}} \qquad 10.14$$

Example 10.4 Two point charges of $q_1 = 6.0$ μC, and $q_2 = 8.0$ μC, are located along the x-axis 2 m apart, as shown in Figure 10.6.

a) What is the work required to bring a third charge $q_3 = -2.0$ μC from infinity to the point p.

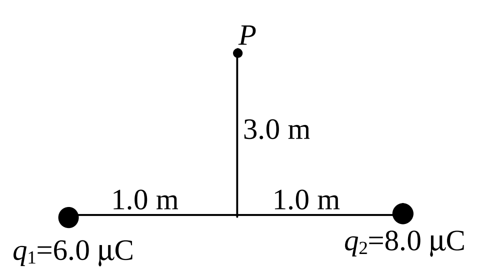


Figure 10.6 Example 10.4.

b) Calculate the electrostatic potential energy of the three charges system.

Solution a) From Equation 10.12, the work required to bring q_3 from infinity to point P is simple the potential at point P multiplied by the charge of q_3. The potential at p is

$$V_p = k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right) = 9\times10^9\left(\frac{6.0\times10^{-6}}{3.2}+\frac{8.0\times10^{-6}}{3.2}\right) = 3.9\times10^4 \text{ V}$$

So the work required is

$$W = q_3 V_P = \left(-2.0\times10^{-6}\right)\left(3.9\times10^{4}\right) = -7.8\times10^{-2}\ \text{J}$$

b) Using Equation 10.13 we have

$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= 9\times10^{9}\left(\frac{(6.0)(8.0)\times10^{-12}}{2.0} + \frac{(6.0)(-2.0)\times10^{-12}}{3.2} + \frac{(8.0)(-2.0)\times1}{3.2}\right.$$

$$= 1.4\times10^{-1}\ \text{J}$$

10.4 ELECTRIC POTENTIAL AND CONDUCTORS

Conductors in electrostatic equilibrium have very important properties mentioned in section 9.8. Another important property concerning the electric potential is

The electric potential inside any conductor is constant and equal to the potential on its surface.

To prove this property we have

$$V_f - V_i = -\int_i^f \boldsymbol{E}\cdot d\boldsymbol{l}$$

If the initial and the final points lie inside a conductor, and using the fact that E=0 inside a conductor, we conclude that the electric

potential is constant inside the conductor. Furthermore, since $\boldsymbol{E}$ is always normal to d$\boldsymbol{l}$ on the surface, the electric potential is also constant on the surface of the conductor.

Consider again a conducting sphere of radius R and charge Q. The electric potential at a point inside the sphere is, from Equation 10.6

$$V = -\int_{\infty}^{r} \boldsymbol{E} \cdot d\boldsymbol{r} = -\int_{\infty}^{R} \boldsymbol{E}_{\text{out}} \cdot d\boldsymbol{r} - \int_{R}^{r} \boldsymbol{E}_{\text{in}} \cdot d\boldsymbol{r}$$

But E_{in}=0 and $\boldsymbol{E}_{\text{out}} = K\dfrac{Q}{r^2}\hat{\boldsymbol{r}}$, so we get

$$V = K\frac{Q}{R}$$

which is the value of the potential at the surface of the conductor.

If two or more conducting objects are connected by a conducting wire, the conductors are no longer separate but can be considered as a single conductor. This means that the electric charges will transfer from the conductor of higher potential to that of lower potential until the equilibrium condition is achieved. Therefore, if two or more conductors are connected and equilibrium is achieved, they must be at the same electric potential.

In analogue with the electric filed lines, the electric potential can be represented by equipotential surfaces. **A surface with all its points are at the same electric potential is called**

equipotential surface. From this definition, it follows that no work is done by the electric field in moving a charged particle between tow points on the same equipotential surface. This means that the electric filed lines must be perpendicular to the equipotential surfaces. Figure 10.- shows the equipotential surfaces for some common charge distributions. The surface of any conductor forms an equipotential surface. As a consequence of this fact, **charge tends to accumulate at sharp points**, as will be proved in example 10.8.

Example 10.8 Two conducting charged spheres with radii R_1 and R_2 are separated by a distance much larger than the radius of either sphere. The two spheres are connected by a conducting wire.
a) Find the ratio of the final charges on the spheres.
b) Find the ratio of the charge densities on the surfaces of the spheres.

Solution Since the spheres are connected by a conducting wire, the potential is the same for both spheres, i.e.,

$$V_1 = V_2$$

$$K\frac{q_1}{R_1} = K\frac{q_2}{R_2}$$

From which it follows that

$$\frac{q_1}{q_2} = \frac{R_1}{R_2}$$

b) The charges are distributed over the surface of the spheres, so we have

$$s_1 = \frac{q_1}{4pR_1^2} \quad \text{and} \quad s_2 = \frac{q_2}{4pR_2^2}$$

Therefore

$$\frac{s_1}{s_2} = \frac{q_1 r_2^2}{q_2 r_1^2}$$

Substituting for the ratio q_1/q_2 given above we get

$$\frac{s_1}{s_2} = \frac{r_2}{r_1}$$

It is clear from this result that the charge density is greatest on the small sphere as expected. Furthermore, since the electric field just outside a conductor is proportional to the charge density, the field is more intense near the smaller sphere.

10.5 CAPACITANCE

Any two conductors with an insulator between them form a **capacitor**. The conductors usually have equal but opposite charges so that the net charge of any capacitor is zero. However, when we said that a capacitor has a charge Q, we mean that one of

the conductors has a charge Q and the other has a charge $-Q$. The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge Q on either conductors to the magnitude of the potential difference, denoted hereafter by V, i.e.,

$$C = \frac{Q}{V} \tag{10.15}$$

The capacitance is always a positive quantity. From Equation 4.1 it is clear that the SI unit of the capacitance C is Coulomb per volt (C/V). This unit is referred as **Farad** (F) in honor of Michael Faraday, that is

$$1F = 1\ C/V$$

Capacitors are very vital elements of almost all electronic devices used to store charges and consequently electrostatic energy. They are essential in circuits we use to tune radio and television transmitters and receivers. They are used also to regulate the output of the electronic power supplies. The microscopic capacitors form the memory bank of computers.

The Parallel Plate Capacitor

The capacitor consists of two parallel metal plates of equal area A separated by a distance d and immersed in vacuum, as shown in Figure

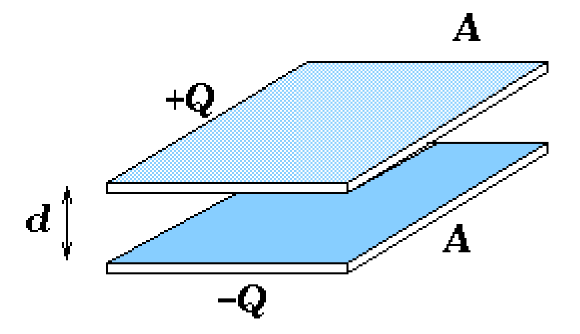


Figure 10.8 A parallel plate capacitor with plate's area A and separation d.

10.8. The two plates have equal but opposite charges, Q and $-Q$. If the separation d is small compared to the size of the plates we can assume the electric field to be uniform in the region between the plates. In this region any point can be considered to be just outside a conductor, and so the magnitude of the electric field of such a configuration, from Equation 9.16, is

$$E = \frac{s}{e_o}$$

with $s = Q/A$ is the charge density on either plate. Using Equation 10.15 with the integral is to be evaluated along a path that starts at one plate and ends on the other, i.e.,

$$V = -\int_{-}^{+} \boldsymbol{E} \cdot d\boldsymbol{l}$$

where + and – refer, respectively to the positive and negative plates of the capacitor. Since $\boldsymbol{E}$ is constant and directed from the positive to the negative plate we can write

$$V = E\int_{-}^{+} dl = Ed$$

$$V = \frac{s}{e_o}d = \frac{Q}{Ae_o}d$$

Substituting this value into Equation 4.1 we obtain

$$C = \frac{Q}{V} = \frac{Q}{Qd/Ae_{o}}$$

or

$$C = e_{o}\frac{A}{d} \qquad 10.16$$

This means that the capacitance depends only on the geometry of the capacitor; directly proportional to the area of the plates and inversely proportional to the separation with the permittivity ε_o stands for the proportionality constant. We will see later that if the medium between the plates is not vacuum this constant should be multiplied by a factor.

In circuits the capacitors and the batteries are represented as shown in Figure 10.9. We can charge a capacitor by connecting it across the terminal of a battery as shown in Figure 10.10. In doing so electrons transfer from the plate that is connected to the positive terminal of the battery (the longer line of the battery symbol) to the plate that is connected to the negative terminal of the battery. This process continues for a short time until the potential difference across the capacitor becomes equal to the potential difference of the battery.

Example 10.9 A parallel plate capacitor of separation 2.00 mm and plate area of 0.500 m^2 are connected to a 800.-V battery. Calculate (a) the capacitance of the capacitor, (b) the charge on the capacitor

Solution (a) Using Equation 4.2 we have

$$C = e_o \frac{A}{d} = 8.85\times10^{-12}\frac{0.500}{2.00\times10^{-3}} = 2.21\text{pF}$$

(b) From Equation 4.1 we get

$$Q = CV = \left(2.21\times10^{-9}\right)(800) = 1.77 m\text{C}$$

10.6 COMBINATIONS OF CAPACITORS

It occurs sometimes that a group of capacitors are connected in the same circuit. The combination is often replaced by a single equivalent capacitor with a capacitance equal to the capacitance of

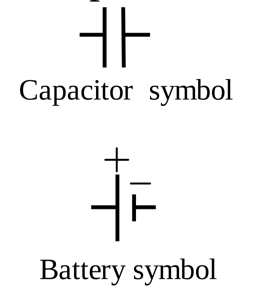


Figure 10.9 Circuit symbols for capacitors and batteries.

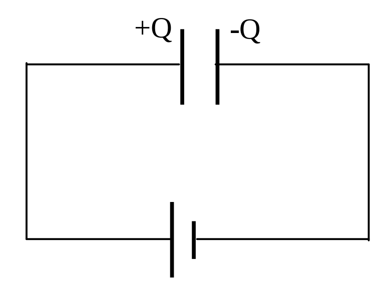


Figure 10.10 Capacitor charged to a battery.

the whole combination. Two basic combinations of capacitors are discussed here: The parallel, and the series combinations.

I-Parallel Combination

If two capacitors are connected as shown in Figure 10.11 we say that the two capacitors are connected in parallel. In such a combination, the right plates are connected, by a conducting wire, together to form an equipotential surface (They have the same potential). The other two plates, the left plates, form a another equipotential surface. Therefore, the potential difference across the two capacitors are the same and equal to the potential difference across an equivalent capacitor replacing the two capacitors, then we have

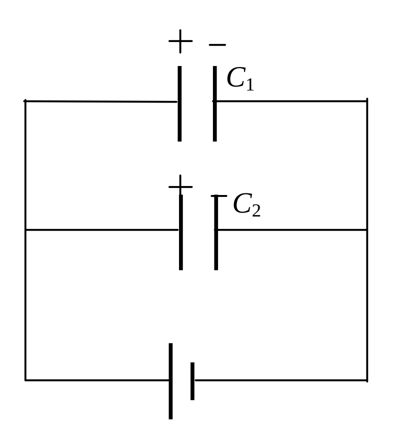


Figure 10.11 Two capacitors are connected in parallel.

$$V_{\mathrm{eq}} = V_1 = V_2 \qquad 10.17$$

where V_{eq} stands for the potential difference across the equivalent capacitor. The charge stored by this equivalent capacitor Q_{eq} is equal to the sum of the charges stored by each capacitor, i.e.,

$$Q_{\mathrm{eq}} = Q_1 + Q_2 \qquad 10.18$$

Using Equation 10.15, the last equation can be written as

$$C_{\mathrm{eq}} V_{\mathrm{eq}} = C_1 V_1 + C_2 V_2$$

Using Equation 10.17 we finally obtain

$$C_{\text{eq}} = C_1 + C_2 \qquad 10.19$$

This can be extend to any number of capacitors, that is

Capacitors are said to be connected in parallel if the potential across each one is the same and equal to the potential across an equivalent capacitor. The equivalent capacitance of the combination is equal to the sum of the capacitance of each capacitor.

II Series Combination

Figure 10.12 illustrates two capacitors connected in series. When the battery is connected, electrons transfer from the left plate of C_1 to the right plate of C_2 through the battery. Thus, the left plate of C_1 acquires a positive charge while the right plate of C_2 acquires an equal negative charge. As the other two plates, enclosed by the dashed line in Figure 10.12, form an isolated conductor, electrons are attracted to the left end leaving the right end with an excess positive charges. This means that the battery induces a charge on the isolated conductor. It is clear here that the charges on each

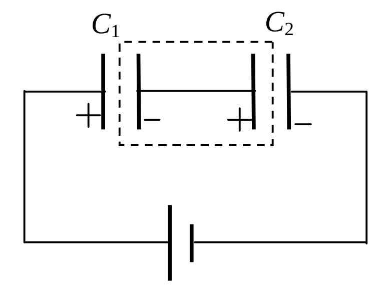


Figure 10.12 Two capacitors are connected in series.

capacitor must be the same and equal to the charge on an equivalent capacitor replacing the two capacitors, that is,

$$Q_{eq} = Q_1 = Q_2 \qquad 10.20$$

and

$$V_{ab} = V_{eq} = V_1 + V_2 \qquad 10.21$$

Again using Equation 10.15 we write

$$\frac{Q_{eq}}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

But, from Equation 10.20, the nominators cancel yielding

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad 10.22}$$

This also can be generalized to any number of capacitors connected in series. Thus

> **Capacitors are said to be connected in series if the charge on each one is the same and equal to the charge on an equivalent capacitor. The reciprocal of the equivalent capacitance equals the sum of the reciprocals of the capacitance of each capacitor.**

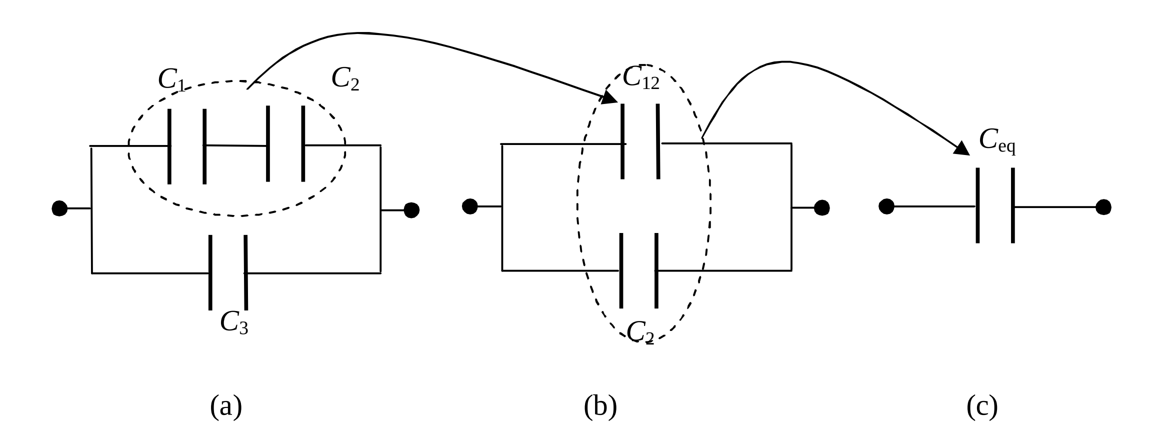


Figure 10.13 Example 10.10.

Example 10.10 Three capacitors, $C_1 = 6.00 m\text{F}$, $C_2 = 6.00 m\text{F}$, and $C_3 = 4.00 m\text{F}$ are connected as shown in Figure 10.13. (a) What is the equivalent capacitance of the combination? (b) If the combination is connected to a battery of 12.0 V, calculate the potential difference across, and the charge on each capacitor.

Solution (a) Using the series and parallel rules, we reduce the combination step by step until we reach a single capacitor, considered to be the equivalent capacitor, as indicated in the figure. C_1 and C_2 are in series. From Equation 10.22, their equivalent capacitance is

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = 3.00 mF$$

As Figure 10.13b shows, C_{12} and C_3 are connected in parallel. Their equivalent capacitance, from Equation 10.19 is

$$C_{eq} = C_{123} = C_{12} + C_3 = 3.00 + 4.00 = 7.00 mF$$

(b) Since C_{12} and C_3 are connected in parallel then

$$V_{12} = V_3 = V_{eq} = 12.0\text{V}$$

Now from Equation 10.15, we have

$$Q_3 = C_3 V_3 = (12.0)(4.00)) = 48.0m\text{C}$$

and

$$Q_{12} = C_{12} V_{12} = (12.0)(3.00)) = 36.0m\text{C}$$

This same charge exists on C_1 and on C_2 due to the series combination between them, i.e.,

$$Q_1 = Q_2 = Q_{12} = 36.0m\text{C}$$

To find the potential difference across C_1 and C_2 we have

$$V_1 = \frac{Q_1}{C_1} = \frac{36}{6.0} = 6.00\text{V}, \text{ and}$$

$$V_2 = \frac{Q_2}{C_2} = \frac{36}{6.0} = 6.00\text{V}$$

10.7 ENERGY STORED IN A CHARGED CAPACITOR

As mentioned in the previous section, in charging a capacitor electrons are transferred from one plate to the other building up a potential difference across the capacitor. This means that a work is required to charge a capacitor and this work is stored as a potential energy in the capacitor.

To calculate the potential energy *U* stored in a charged capacitor, we consider a parallel plates capacitor that is initially uncharged. Suppose that *q* is the charge built up on the capacitor at some instant during the charging process. The potential difference across the capacitor at that instant is, from Equation 10.15, $v = q/C$, with C is the capacitance of the capacitor. The work required to transfer a small charge *dq* is therefore

$$dW = vdq = \frac{q}{C}dq$$

The total work required to charge a capacitor from uncharged situation (*q*=0) to a final charge *Q* is, thus

$$W = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C} \qquad 10.23$$

This total work will be stored in the capacitor as potential energy. Using Equation 10.15 we can write

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV \qquad 10.24$$

Example 10.11 Referring to the previous example Find (a) The energy stored by each capacitor. (b) The energy stored by the group.

Solution (a) The charge on each capacitor if found in the previous example. Using Equation 10.24 we have

$$U_1 = \frac{Q_1^2}{2C_1} = \frac{(36.0)^2}{12.0} = 108\,\text{J}$$

$$U_2 = \frac{Q_2^2}{2C_2} = \frac{(36.0)^2}{12.0} = 108\,\text{J}$$

$$U_3 = \frac{Q_3^2}{2C_3} = \frac{(48.0)^2}{8.00} = 288\,\text{J}$$

(c) Using again Equation 10.24 we have

$$U_{\text{eq}} = \frac{Q_{\text{eq}}^2}{2C_{\text{eq}}} = \frac{(7.00\times 12.0)^2}{2\times 7.00} = 504\,\text{J}$$

Note that $U_{eq} = U_1 + U_2 + U_3 + U_4$

10.8 CAPACITORS WITH DIELECTRIC

A **dielectric** is an insulating material such as rubber, glass, or plastic. It is found that when a dielectric material is inserted between the plates of a capacitor, its capacitance increases by a numerical factor κ called the **dielectric constant** of the material, that is

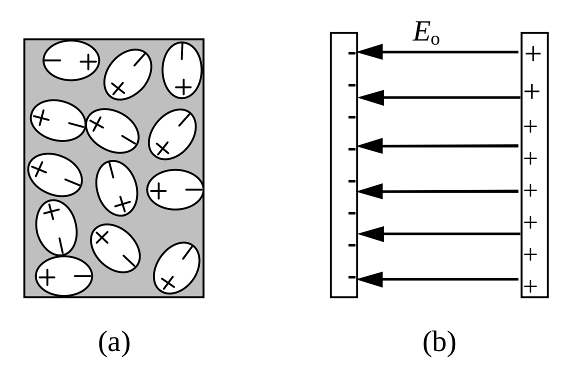
$$C = kC_o \qquad 10.25$$

where C and C_o are the capacitance with and without the dielectric respectively. Since C is always greater than C_o, the dielectric constant κ must be greater than unity. Table 10.1 gives the dielectric constants for some materials.

Let us now explain what happen when a dielectric material is inserted between the plates of an isolated capacitor. Suppose that the dielectric is a polar material (has a permanent electric dipole). The electric field of the capacitor E_o exerts a torque on the dipoles of the material so that it tends to rotates

Table 10.1 Dielectric constants and dielectric strengths for some materials at room temperature. The dielectric strength is the maximum electric field before breakdown (charge flow) occurs.

Mater	[illegible]	Dielectric strength (V/m)
Vacuu	[illegible]	3 $\times 10^6$
Air (d	[illegible]	24 $\times 10^6$
Paraffi	[illegible]	10 $\times 10^6$
Polystyrene	2.6	24 $\times 10^6$
Paper	[illegible]	15 $\times 10^6$
Quartz	4.3	8 $\times 10^6$
Oil	4	12 $\times 10^6$
Glass	5	14 $\times 10^6$
Rubber	6.7	12 $\times 10^6$
Porcelain	6-8	5 $\times 10^6$
Nylon	3.4	14 $\times 10^6$
Water	80	



E_o

E'

(c)

Figure 10.14 (a) A dielectric slab with its molecules are randomly oriented. (b) A charged capacitor without a dielectric. E_o is the electric field inside the capacitor without the dielectric. (c) The dielectric being inserted between the charged capacitor. The net electric field is now E_o-E.'

these dipoles into the direction of $\boldsymbol{E}_o$. In this case the material is said to be **polarized**. As a result of this polarization, polarization charges are produced at the faces of the dielectric as shown in Figure 10.14.(the charges inside the dielectric cancel each other). Note that the positive charges are near the negative plate and the negative charges are near the positive plate. These charges create an electric field $\boldsymbol{E}'$ opposite to $\boldsymbol{E}_o$. The net electric filed inside the conductor is therefore

$$E = E_o - E' \qquad 10.26$$

That is, a reduction is occurred in the electric field.

If the dielectric is not polar, it will acquires a nonpermanent, induced dipole moment when placed in an external electric field. The effect of the external electric field is to stretch the molecule and therefore, the centers of the positive and the negative charges are slightly separated. These induced dipoles tend to align with the external electric field so that polarization charges formed at the faces of the dielectric and so a reduction of the electric field is also achieved.

Since $V = Ed$, the potential difference across the capacitor decreases, consequently. Now from Equation 10.15 and since the charge is constant, we conclude that the capacitance is increased by insertion of a dielectric material between the conductors of a capacitor

Example 10.12 A parallel plate capacitor, of capacitance $C_o = 8.0\,\mu F$ is charged by a 36-V battery. The battery is then removed and a dielectric slab of dielectric constant $\kappa = 3.7$ is inserted between the plates of the capacitor.

a) What is the capacitance after inserting the dielectric?
b) Find the energy stored in the capacitor before and after the slab is inserted.

Solution (a) From Equation 10.25 we have

$$C = kC_o = (3.6)(8.0mF) = 29mF$$

(b) The energy stored in the capacitance before inserting the dielectric is

$$U_o = \tfrac{1}{2}C_oV_o^2 = \tfrac{1}{2}(8.0mF)(36)^2 = 5.2\times10^{-3}\,\mathrm{J}$$

After removing the battery the charge on the capacitor will not be changed even if the capacitance is increased by inserting the dielectric. This unchanged charge is

$$Q_o = eC_o = (36\mathrm{V})(8.0mF) = 290mC$$

Now, the energy stored after introducing the dielectric is

$$U = \frac{Q_o^2}{2C} = \frac{\left(288\times10^{-6}\right)^2}{2\left(28.8\times10^{-6}\right)} = 1.4\times10^{-3}\,\mathrm{J}$$

The difference in the energy $U_o - U$ can be explained as the work done by the capacitor.

CHAPTER 11

CURRENT AND RESISTANCE

11.1ELECTRIC CURRENT

Whenever electric charges of the same sign move, we have what is called electric current.

$$I = \frac{dQ}{dt} \qquad 11.1$$

The SI unit of current is the ampere, abbreviated (A) with

1 A = 1 C/s

Although current is a scalar quantity, a direction is always specified to the current. The direction of the current is taken, by convention, to be the direction of motion of positive charges. Thus, the electric current is directed from the points of higher potential to the points of lower potential.

Electric current per unit area in the direction of the positive charges motion is known as the **current density *J***. In another word, $\boldsymbol{J}$ is related to the current I through

$$dI = \boldsymbol{J} \cdot d\boldsymbol{A} \qquad 11.2$$

If the current density is uniform and $\boldsymbol{J}$ and $d\boldsymbol{A}$ are parallel, the total current through any surface A is given by

$$I = JA \qquad 11.3$$

The SI unit of J is A/m^2.

Example 11.1 A current of 3.2 mA is flowing in a wire. Calculate the number of electrons passing a given point on the wire in one second.

Solution Let us first calculate the total charge passing the point in one second. From Equation 11.2 we have

$$Q = It = 3.2\times10^{-3}\,C$$

To find the number of electrons we have to divide by the charge of each electron, that is

$$Number = \frac{Q}{e} = \frac{3.2\times10^{-3}}{1.6\times10^{-19}} = 2.0\times10^{16}\text{ electrons}$$

This a huge number, of course.

11.2OHM'S LAW AND RESISTANCE

To have a charge flow between two points, a potential difference across these two points must be maintained. It is found that, for most materials, *the current in a wire is linearly proportional to the potential difference across this wire*. This statement is known as **Ohm's law** which can be expressed as

$$V = RI \qquad 11.4$$

The proportionality constant R is called the **resistance** of the wire. It measures the degree to which the wire opposes the current

through it. The SI unit of resistance is Volt per Ampere (V/A) or **Ohm** (Ω), that is

$$1\Omega = 1\text{V/A}$$

The wire with considerable resistance is called **resistor**. In circuit diagram, the resistor is representd by the symbol .

The resistance R is found to be directly proportional to the length of the resistor and inversely proportional to its cross-sectional area, that is.,

$$R = r\frac{L}{A} \qquad 11.5$$

The proportionality constant r is called the **resistivity** of the material of which the wire is made. Table 11.1 gives the resistivities of some commom materials.

The reciprocal of the resistivity is called the **conductivity** σ, that is

$$s = \frac{1}{r} \qquad 11.6$$

The SI unit of ρ and of σ are, respectively, Ω.m, and $(\Omega.\text{m})^{-1}$.

Consider now a

Table 11.1 Resistivity and temperature coefficients at 20°C.

Material	**Resistivity (W.m)**	**Temperature Coefficient $(C^o)^{-1}$**
Silver	1.59×10^{-8}	6.1×10^{-3}
Copper	1.68×10^{-8}	6.8×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.65×10^{-8}	4.3×10^{-3}
Tungsten	5.60×10^{-8}	4.51×10^{-3}
Iron	9.71×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
Nichrome	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	$10^{9} - 10^{12}$	
Rubber	$10^{13} - 10^{15}$	

conductor of length L and cross-sectional area A that maintained at a potential difference V. The potential difference V is related to the electric field, assumed to be uniform, through the relation

$$V = El \qquad 11.7$$

Substituting for V from Equation 11.4 we get

$$IR = EL \qquad 11.8$$

Substituting for I from Equation 11.3 and for R from Equation 11.5, we obtain

$$\boldsymbol{J} = \frac{1}{r} E = s\boldsymbol{E} \qquad 11.9$$

That is, the ratio of the current density and the electric field is constant. This statement is known as **Ohm’s law**. Equation 11.4 and Equation 11.9 are two forms of Ohm’s law; the first refers to a specific piece of material, while the second refers to a general property of the material.

Example 11.2 Calculate the resistance of a rod of silver of length 30.0 cm and of cross-sectional area of 2.00×10^{-4} m^2. Repeat the calculation for a similar carbon rod.

Solution Using Table 11.1 and Equation 11.7 we have

$$R = r\frac{L}{A} = \left(1.59\times10^{-8}\right)\left(\frac{0.3}{2\times10^{-4}}\right) = 2.39\times10^{-5}\,\Omega$$

Similarly for the carbon rod we have

$$R = r\frac{L}{A} = \left(3.5\times10^{-5}\right)\left(\frac{0.3}{2\times10^{-4}}\right) = 5.25\times10^{-2}\,\Omega$$

11.3 CAUSE OF RESISTANCE AND TEMPERATURE DEPENDENCE

At room temperatures, collision of electrons with the vibrating atoms is the prime cause of metal resistance. Imperfections of lattice also contribute into resistance, although their contribution in pure metals is negligible**.**
At higher temperatures, the atoms are vibrating more rapidly and arranged in a less regular pattern. So they expected to be more active in the collision process. It is found that the resistivity for metals increases with temperature according to

$$r = r_o[1 + a(T - T_o)] \qquad 11.10$$

Where r and r_o are, respectively, the resistivity at temperature T and T_o. The constant a is called **the temperature coefficient of resistivity**. Values for a in some materials are given in Table 11.1. Since the resistance of a wire is proportional to the resistivity according to Equation 11.5, the variation of resistance with temperature can be written as

$$R = R_o[1 + a(T - T_o)] \qquad 11.11$$

Note that for semiconductors α is negative. This because some electrons that are normally not free at room temperature becomes free at higher temperatures. So resistivity decreases as temperature increases.

Example 11.3 The resistance of a platinum resistance thermometer is measured at 20.0^o C to be 164 Ω. When placed in a particular solution, the resistance is measured to be 192 Ω. What is the temperature of the solution?

Solution Using Table 11.1 and Equation 11.11 we have

$$T = T_o + \frac{R - R_o}{aR_o}$$

$$= 20.0 + \frac{192 - 164}{(3.927 \times 10^{-3})(164)} = 63.5^o \text{C}$$

11.4 ELECTROMOTIVE FORCE

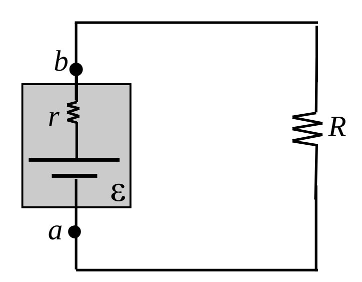


Figure 11.1 A circuit diagram for a source of *emf* with internal resistance *r* and load resistor *R*.

Consider the circuit shown in Figure 11.1. Current will be established through this resistor *R* if a potential difference is maintained across its ends. If the end *b* is at higher potential than the end *a*, then charge will move through the resistor from *b* to *a*. For the current to circulate around a closed circuit, the charge must jump from *a* to *b*. This means that we need a device that capable of pumping charge from lower potential to higher potential. The function of such a device is called **electromotive force**, abbreviated ***emf***, and denoted by the symbol ε. The battery and the generator are common *emf* devices. A source of emf can be considered as a charge pump that pumps charges in a direction opposite to the

electrostatic force inside the source. It is exactly like a water pump that pushes water from lower to a higher level opposite to the gravitational force.

The resistance r is called the internal resistance of the battery, and R is called the load resistor. We shall assume that the connecting wire have no resistance. Any positive charge moving from a to b will gain a potential ε as it passes from the negative terminal to the positive terminal of the battery. However, it will lose a potential Ir as it passes through the internal resistor, where I is the current in the circuit. Thus, the terminal voltage of the battery, $V = V_b - V_a$, is given by

$$V = e - Ir \qquad 11.12$$

From this Equation it is clear that the emf is equal to the terminal voltage of a battery in an open circuit, that is, when the current is zero. As the connecting wires have no resistance we conclude that the voltage V must also equal the potential across the load resistor R, that is

$$V = IR \qquad 11.13$$

Combining Equations 11.12 and 11.13, and solving for I, we get

$$I = \frac{e}{R + r} \qquad 11.14$$

Example 11.4 A battery has an emf of 12.0 V and internal resistance of 0.100 Ω. Its terminal is connected to a load resistance of 4.00 Ω. Find the current in the circuit and the terminal voltage of the battery.

Solution Using Equation 11.14 we have

$$I = \frac{e}{R+r} = \frac{12.0}{4.00+0.100} = 2.93\,\mathrm{A}$$

The terminal voltage is, from Equation 11.12

$$V = e - Ir = 12.0 - (2.93)(0.100) = 11.7\,\mathrm{V}$$

Note that the terminal voltage is the same as the voltage across the load resistance, that is

$$V = IR = (2.94)(4.00) = 11.7\,\mathrm{V}$$

11.5 ELECTRIC POWER

Consider the circuit shown in Figure 11.2 where a battery, with emf ε, and negligible internal resistance, is connected to a resistor *R*. A positive charge *dq* will gain potential energy as it moves from point *a* to point *b* through the battery and loses the same amount of potential energy as it moves from point *c* to point *d* through the resistor. This

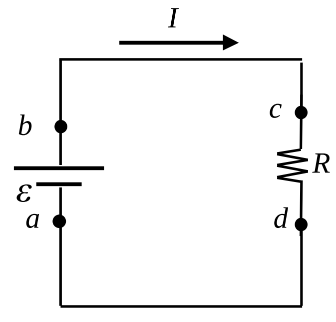


Figure 11.2 Charges gain energy as it move from *a* to *b* through the battery, while they lose this energy as it moves from *c* to *d* through the resistor.

mount is given by

$$dU = dqe = dqV \qquad 11.15$$

where V is the potential difference across the resistor. Since the power is defined as the rate of change of potential energy, we obtain

$$P = \frac{dU}{dt} = \frac{dq}{dt}V = IV \qquad 11.16$$

Using the formula $V = IR$, we can express the power in three alternative forms

$$P = IV = I^2R = \frac{V^2}{R} \qquad 11.17$$

Recalling from chapter 5 that the SI unit of power is watt and using the first part of Equation 11.16 you can justify the use of **Kilowatt-Hour** (KW.Hr) as the commercial unit of electric energy.

Example 11.5 You are given an electric heater made of nichrome wire of resistance 25 Ω. Find the current carried by the wire and the power of the heater if it is connected to 220 V source.

Solution The current can be found from

$$I = \frac{V}{R} = \frac{220}{25} = 8.8\text{A}$$

and the power can be found from Equation 11.16 as

$$P = I^2 R = (8.8)^2 (25) = 1.9\,\text{KW}$$

Example 11.6 A light bulb of power 100. W is operating to 220. V. Find its resistance and the current in the bulb.

Solution Using the relation $P = \frac{V^2}{R}$, we get

$$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\,\Omega$$

Now from Ohm's law we have

$$I = \frac{V}{R} = \frac{220}{484} = 0.454\,\text{A}$$

We can find the current from the equation

$$I = \frac{P}{V} = \frac{100}{220} = 0.454\,\text{A}$$

CHAPTER 12

DIRECT CURRENT CIRCUITS

12.1RESISTORS IN SERIES AND IN PARALLEL

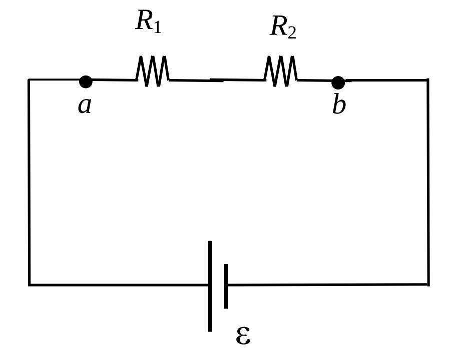


Figure 12.1 Series connection of two resistors. The current in each resistor is the same.

When two resistors are connected together as shown in Figure 12.1 we said that they are connected in series. As it is clear from the figure, any charge that flows through R_1 must equal the charge that flows through R_2, that is

$$I_{eq} = I_1 = I_2$$

Since the potential difference between a and b equals the sum of the potential drop across each resistor we have

$$V_{eq} = V_1 + V_2$$

$$V_{eq} = IR_1 + IR_2 = I(R_1 + R_2) \qquad 12.1$$

where V_{eq} is the potential drop across the equivalent resistor. Therefore we conclude that

$$R_{eq} = R_1 + R_2 \qquad 12.2$$

For more than two resistors connected in series we have

$$R_{eq} = R_1 + R_2 + R_3 + \ldots \qquad 12.3$$

Now consider the two resistors connected as shown in Figure 12.2. The potential drops across R_1 and R_2 are equal and must equal to the potential drop across any equivalent resistor connected between a and b, that is

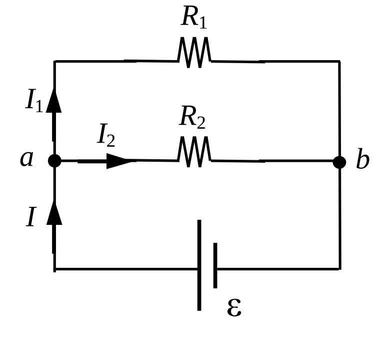


Figure 12.2 Parallel connection of two resistors. The potential difference across each resistor is the same.

$$V_{eq} = V_1 = V_2 \tag{12.4}$$

If I_1 and I_2 are the currents passing through R_1 and R_2, respectively, then the net current of the circuit is

$$I = I_1 + I_2 \tag{12.5}$$

Using Ohm's law and Equation 12.4, we get

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{12.6}$$

In general if more that two resistors are connected in parallel, then we have

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \mathbf{L} \tag{12.7}$$

Household circuits are always connected in parallel so that if one device is switched off the others remains on. The operating voltage of all the devices is the same in such a connection.

Example 12.1 Three resistors are connected as shown in Figure 12.3.
(a) What is the equivalent resistance of the circuit?
(b) If the *emf* of the battery is 56 V, find the current through each resistor

Solution **(a)** The circuit can be reduced, step by step, to a single equivalent resistance as shown in Figure 12.3. The 8.0-Ω and the 2.0-Ω are connected in parallel , and so they can be replaced by an equivalent resistor of 1.6 Ω, using Equation 12.6. This resistor is connected in series with the 4.0-Ω resistor. The equivalent resistance of the circuit is then

$$R_{eq} = 4.0 + 1.6 = 5.6\Omega$$

(b) Since the 4-Ω and the 1.6-Ω are connected in series, they have the same current I_1, which must equal to the current of the battery. Using Ohm's law we get

$$I_1 = \frac{e}{R_{eq}} = \frac{56}{5.6} = 10.\text{A}$$

Now the potential difference across the 1.6-Ω is

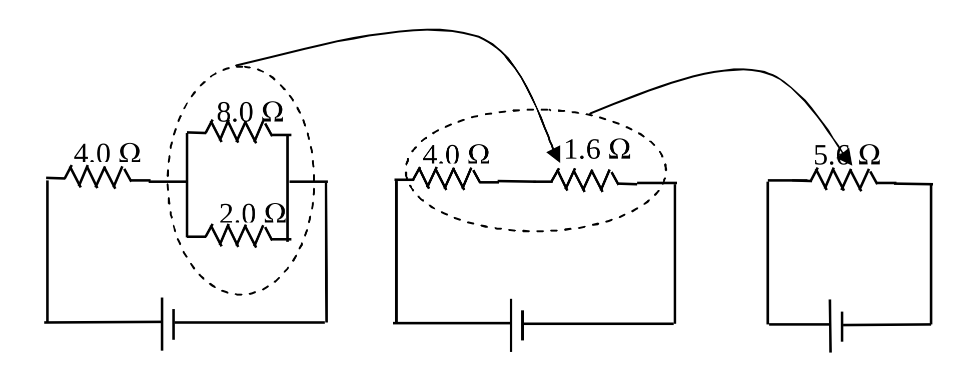


Figure 12.3 Example 12.1

$$V_{bc} = I_1 R = 10.(1.6) = 16\,\text{V}$$

This potential difference is the same across the 8.0-Ω and the 2.0-Ω resistors due the parallel connection between them. So, we can find the current I_2 passing through the 8.0-Ω resistor as

$$I_2 = \frac{V}{8.0} = \frac{16}{8.0} = 2.0\text{A}$$

and the current I_3 through the 2.0-Ω resistor as

$$I_3 = \frac{V}{2.0} = \frac{16}{2.0} = 8.0\,\text{A}$$

Example 12.2 What is the current passing through the 6.0-Ω resistor in the circuit shown in Figure 12.4?

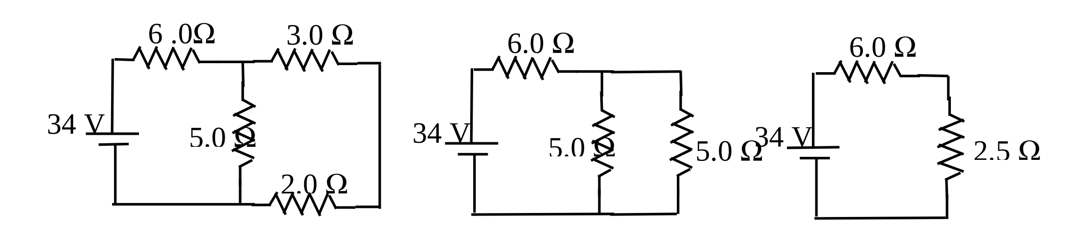


Figure 12.4 Example 12.2

Solution It is clear that the current I passing through the 6.0-Ω resistor must pass through the battery. Therefore, we can write

$$I = \frac{e}{R_{eq}}.$$

Now to find R_{eq} we have to reduce the circuit step by step as we did in the previous example. To do so we have to note that the 2.0-Ω and the 3.0-Ω resistors are connected in series. Their equivalent resistor is connected in parallel with the 5.0-Ω resistor. The equivalent resistor of the three resistors is connected in series with the 6.0-Ω resistor. The equivalent resistance of the circuit R_{eq} is then $R_{eq} = 8.5\Omega$. So we have

$$I = \frac{34}{8.5} = 4.0\text{A}$$

6.2 KIRCHHOFF'S RULES

The series-parallel combinations of resistors are not capable of handling some complex circuits. Gustav Robert Kirchhoff (1824-1887) developed two rules for analyzing such circuits:

I- The sum of the currents entering any junction must equal the sum of the currents leaving that junction. (A junction is any point in a circuit where a current can split)

II- The algebraic sum of the potential differences across all the elements around any loop must be zero.

The first rule is an application of the conservation of charge principle, while the second rule is an application of the conservation of energy principle.

To apply the second rule we should know the following two remarks:
1- The change in potential through any resistor is negative for a move in the direction of the current and positive for a move opposite to the direction of the current. This is because the current through a resistor moves from the end of higher potential to that of lower potential.
2- The change in potential through an ideal battery is positive for a move from the negative to the positive terminal of the battery and negative for a move in the opposite direction.

Strategy for solving problems using Kirchhoff's rules:

1- Draw a circuit diagram and label all quantities, known and unknown.
2- Assign a direction for the current in each part of the circuit. Do not bother if your guess of current direction is incorrect; the result will have a negative value.
3- Apply the first Kirchhoff's rule to any junction in the circuit. In general this rule is used one time fewer than the number of junctions in the circuit.
4- Choose any closed loop in the network, and designate a direction (clockwise or counterclockwise) to traverse the loop.
5- Starting from one point in the loop, go around the loop in the designated direction. Sum the potential differences across all the elements of the chosen loop to zero. In doing so you should note the two remarks discussed above, that is, the potential difference across an emf is $+\varepsilon$ if it is traversed from the negative to the positive terminal and $-\varepsilon$ if traversed in the opposite direction. The potential difference across any resistor is $-IR$ if this resistor is traversed in the direction of the assumed current and $+IR$ if traversed in the opposite direction.
6- Choose another loop and repeat the fifth step to get a different equation relating the unknown quantities. Continue until you have as many equations as unknowns.
7- Solve these equations simultaneously for the unknowns.

Example 12.3 In the circuit shown in Figure 12.5, find the current in each resistor.

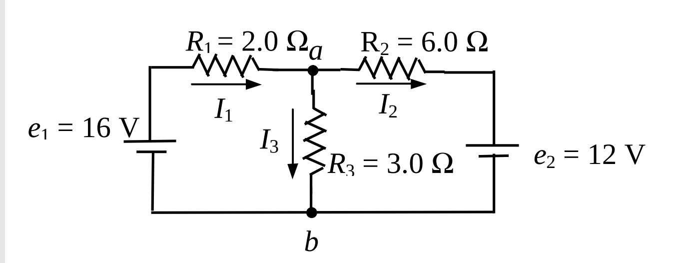


Figure 12.5 Example 12.3

Solution Let I_1, I_2, and I_3 be the currents through the resistors R_1, R_2, and R_3, respectively. The directions of the currents are assigned arbitrary as shown in Figure 12.5. As it clear from the circuit there are two junctions, *a* and *b*, and two loops, the left loop and the right loop. Applying Kirchhoff's first rule to the junction *a* we get

$$I_1 = I_2 + I_3 \tag{1}$$

Note that the same equation will be got if rule is applied to junction *b*. Now, we apply Kirchhoff's second rule to the left loop and traverse the loop in the clockwise direction, obtaining

$$e_1 - I_1R_1 - I_3R_3 = 0$$

$$16 - 2.0I_1 - 3.0I_3 = 0 \tag{2}$$

We need three equations to solve for the three unknowns. To obtain the third equation we traverse the right loop in the counter-clockwise direction. Kirchhoff's second rule then gives

$$e_2 + I_2R_2 - I_3R_3 = 0$$

$$12 + 6.0 I_2 - 3.0 I_3 = 0 \qquad (3)$$

Substituting for I_1 from Equation (1) into Equation (2) we get

$$16 - 2.0 I_2 - 5.0 I_3 = 0 \qquad (4)$$

Solving Equations (3) and (4) for I_2 and I_3 we get

$$I_2 = 0.5\text{A}, \text{ and } I_3 = 3.0\text{A}$$

Substituting these two values into Equation (1) we get

$$I_3 = 3.5\text{A}$$

Example 12.4 Three ideal batteries and three resistors are connected as shown in Figure 12.6.
a) Find the current in each resistor.
b) What is the current through the middle battery?
c) Calculate the potential difference $V_b - V_a$.

Figure 12.6 Example 12.4.

Solution Let us label the currents in each branch of the circuit as I_1, I_2, and I_3 with the assumed directions shown in Figure 12.6. Note that the current I_1 that pass through R_1 must pass through R_3.
a) If we apply Kirchhoff's second rule to the left loop traversing it clockwise we get

$$e_1 - I_1R_1 - e_2 - I_1R_3 = 0$$

$$14 - 4.0I_1 - 8.0 - 2.0I_1 = 0$$

From which we find $I_1 = 1.0\text{A}$
Now, we choose the right loop and traverse it in the clockwise direction. Kirchhoff's second rule then gives

$$e_2 - I_2R_3 - e_3 = 0$$

$$8.0 - I_2 - 10. = 0$$

or $\quad I_2 = 2.0\text{A}$

b) The current I_2 that pass thought the middle battery can be obtained by applying Kirchhoff's first rule to the junction a. This gives

$$I_1 = I_2 + I_3$$

or

$$I_2 = I_1 - I_3 = 1.0 - 2.0 = -1.0\text{A}$$

The minus sign indicates that I_2 should point up

c) Starting at point *a*, we follow a path toward point *b*, adding potential differences across all the elements we encounter. If we follow the path through the middle battery we obtain

$$V_b - V_a = -8.0 - 2.0 I_1 = -8.0 - 2.0 = -10.\text{V}$$

The minus sign here means that $V_a > V_b$. Try to follow another paths from a to b to verify that they also give the same result.

12.3The *RC* CIRCUITS

So far we have discussed circuits with time-independent currents. Now we deal with circuits that contain capacitors. In such circuits the current depends on time.

Charging Process

Figure 12.7 shows a capacitor, initially uncharged, connected in series with a resistor. If the switch S is thrown at point 1 at $t = 0$, the capacitor will begin to charge, creating a current in the circuit. Let I be the current in the circuit at some instant during the charging process, and q be the charge on the capacitor at the same instant. Applying Kirchhoff's second rule to the circuit, we obtain

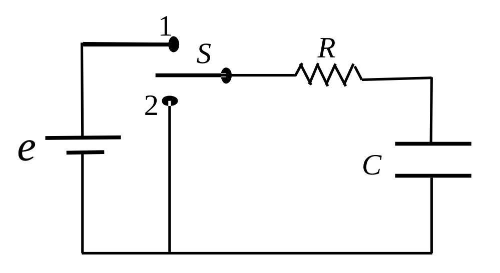


Figure 12.7 When the switch S is at point 1 the capacitor is charged by the battery. When S is at point 2 the capacitor discharge through R.

$$e - IR - \frac{q}{C} = 0 \qquad 12.8$$

The potential difference across the capacitor q/C is negative because there is a drop in potential as we move from the positive

plate of the capacitor to the negative plate. Substituting for I with $I = dq/dt$, in Equation 12.8, and rearranging we obtain

$$\frac{dq}{eC - q} = \frac{dt}{RC} \qquad 12.9$$

Noting that the charge on the capacitor is initially zero, i.e., $q = 0$ at $t = 0$, we can integrate both sides of Equation 12.9 as

$$\int_0^q \frac{dq}{eC - q} = \int_0^t \frac{dt}{RC}$$

$$\ln\left(\frac{eC - q}{eC}\right) = -\frac{t}{RC}$$

$$q = eC\left(1 - e^{-\frac{t}{RC}}\right) \qquad 12.10$$

where e is the base of the natural logarithm. To find the current as a function of time, we differentiate Equation 12.10 with respect to time to get

$$I = \frac{e}{R} e^{-\frac{t}{RC}} \qquad 12.11$$

The quantity RC is called the **time constant**, t, which defined as the time required for the current to decrease to $1/e$ of its initial

value. Equations 12.10 and 12.11, which are plotted in Figure 12.8, tell the following:

1- At $t = 0$, the charge q is zero, as required, and the initial current I_o is

$$I_o = \frac{e}{R} \qquad 12.12$$

that is, the capacitor acts as if it were a wire with negligible resistance (short circuit).

2- As $t \to \infty$(after a long time), the charge has its maximum equilibrium value, Q_m

$$Q_m = eC \qquad 12.13$$

and the current is zero, that is the capacitor acts as it were an open switch (open circuit).

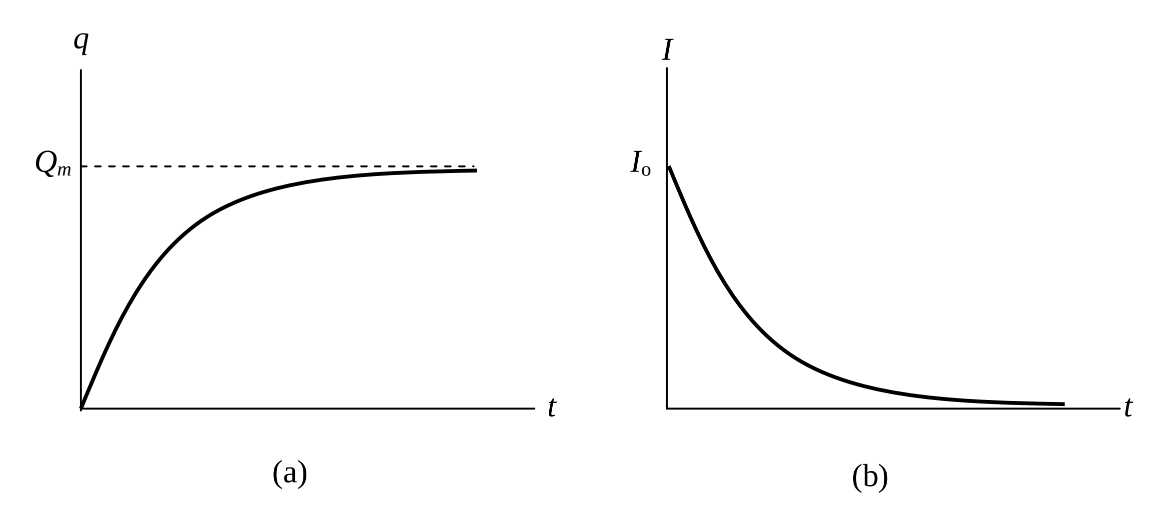


Figure 12.8 (a) The charge versus time in a charging process for *RC* circuit. (b) The current versus time in a charging process for the same *RC* circuit.

Discharging Process

Suppose that the capacitor in Figure 6.17 is now fully charged such that its potential difference is equal to the emf ε. If the switch is thrown to point 2 at a new time $t = 0$, the capacitor begin to discharge through the resistor. Let I be the current in the circuit at some instant during this process, and q be the charge on the capacitor at the same instant. Applying Kirchhoff's rule to the loop, we get

$$\frac{q}{C} - IR = 0 \qquad 12.14$$

Substituting for I with $I = -dq/dt$ (explain the negative sign), and rearrange we obtain

$$\frac{dq}{q} = -\frac{1}{RC}dt \qquad 12.15$$

Using the initial condition, $q = Q_m \text{at} t = 0$ we can integrate the last equation to obtain

$$\int_{Q_{\max}}^{q} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\frac{q}{Q_m} = -\frac{1}{RC}t$$

$$q = Q_m e^{-t/RC} \qquad 12.16$$

The current is the rate of decrease of the charge on the capacitor, that is

$$I = -\frac{dq}{dt} = I_o e^{-t/RC} \qquad 12.17$$

where

$$I_o = \frac{Q}{RC} \qquad 12.18$$

Example 12.5 A 2.0-kΩ resistor and a 8.0-μF capacitor are connected, in series, with a 24-V battery as shown in Figure 12.9. The capacitor is initially uncharged, and the switch S is closed at *t=0*.

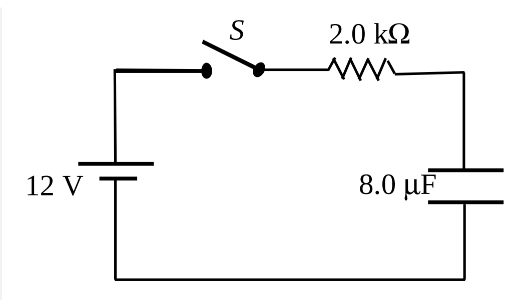
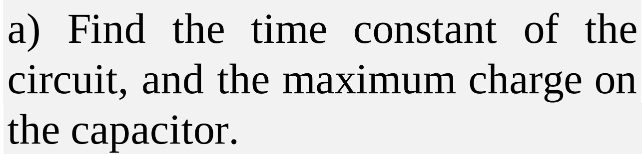


Figure 12.9 Example 12.5.



a) Find the time constant of the circuit, and the maximum charge on the capacitor.

b) What is the time required for the current to drop to half its initial value?

c) After being closed for a long time, the switch is now opened at t=0, what is the time required for the charge to decrease to one-fifth its maximum value.

Solution a) The time constant is

$$t = RC = \left(2.0\times10^{3}\right)\left(8.0\times10^{-6}\right) = 16\text{ms}$$

The maximum charge is , from Equation 12.13,

$$Q_m = eC = (12)\left(8.0\times10^{-6}\right) = 96m\text{F}$$

b) From Equations 6.11 and 6.12 we have

$$I = I_0 e^{-\frac{t}{RC}}$$

To find the time required for the current to drop to half its value, we substitute $I = \frac{1}{2} I_o$ into this equation:

$$\frac{1}{2} I_o = I_o e^{-\frac{t}{RC}}$$

Taking the logarithm of both sides, we have

$$\ln \frac{1}{2} = -\frac{t}{RC}$$

or

$$t = -RC \quad \ln \frac{1}{2} = 11.1\text{ms}$$

c) In the discharging process, the charge varies with time according to

$$q = Q_m e^{-t/RC}$$

Substituting for $q = \frac{1}{5} Q_m$, and taking the logarithm of both sides we get

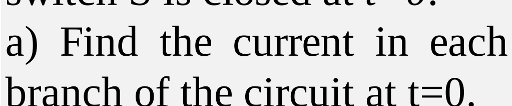
$$\ln \frac{1}{5} = -\frac{t}{RC}$$

or

$$t = -RC \;\; \ln \frac{1}{5} = 26\,\text{ms}$$

Example 12.6 In the circuit shown in Figure 12.10, the capacitor is initially empty and the switch S is closed at *t=0*.

a) Find the current in each branch of the circuit at t=0.



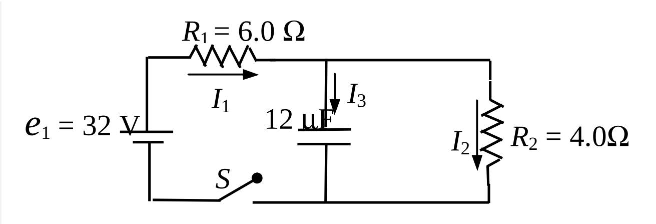


Figure 12.10 Example 12.6.

b) Calculate the maximum charge on the capacitor.

Solution a) At *t=0*, the capacitor is treated as if it were a wire with negligible resistance. Therefore, we have

$$I_2 = 0 \text{ and } I_1 = I_3 = \frac{32}{6.0} = 5.3\,\text{A}$$

b) The maximum charge is attained after a long time ($t \to \infty$). At this time the capacitor is treated as if it were an open switch. So,

$$I_3 = 0 \text{ and } I_1 = I_2 = \frac{32}{(6.0 + 4.0)} = 3.2\text{A}.$$

Applying Kirchhoff’s second rule to right loop we find that the potential difference *V* across the capacitor is

$$V = I_2 R_2 = (3.2)(4.0) = 12\,\text{V}$$

Now the maximum charge is

$$Q = CV = \left(12\times10^{-6}\right)(12.8) = 1.5\times10^{-4}\,mC$$

6.4 ELECTRICAL INSTRUMENTS

The Ammeter It is a device used to measure the current. The current to be measured must pass through the ammeter, that is the ammeter should be connected in series with the current it is to measure. The resistance of an ammeter must be small compared to other resistances in the circuit so as not to alter the current being measured.

The Voltmeter It is a device used to measure the potential difference. To measure a potential difference between two points the terminals of the voltmeter have to be connected, in parallel, between these two points. The resistance of a voltmeter must be large enough not to alter the voltage being measured.

CHAPTER 13

FLUID MECHANICS

Matter is classified into three states; solid state, fliud, and plasma state. The solid state is a substance with definite volume and shape. The second state is the fluids (liquids + gases), which is a substance that can flow. Liquids has a definite volume but no definite shape. Gases have neither definite volume nor definite shape. The third state is the plasma state, which is a gas contains a collection of negatively charged electrons and positively charged ions. Fluid mechanics play an important role in many branches of applied sciences. For example, flight, ship science, meteorology, the influence of wind upon building structures, groundwater seepage and turbines as practical situations in which the dynamics of fluids plays an important role. Indeed there are few real-life situations in which the flow of fluids is not in some way significant.

13.1 DENSITY AND PRESSURE

Density

The density r of a substance is defined as the ratio of mass m to volume V; that is

$$r = \frac{m}{V} \tag{13.1}$$

The density of a homogeneous substance is a constant quantity and it is independent of volume or mass. The unit of density are kg/m^3 in the SI system and g/cm^3 in the cgs system. The densities of some commom substanes are listed in Table 13.1.

Pressure

In general the pressure p is defined as the ratio of normal force F to the area A, i.e.

$$p \equiv \frac{F}{A}, \tag{13.2}$$

Table 13.1 Densities of some substances.

Substance	Density g/cm^3
Aluminum	2.7
Benzene	0.88
Copper	8.9
Glycerin	1.26
Gold	19.3
Ice	0.92
Iron	7.8
Lead	11.3
Platinum	21.4
Silver	10.5
Steel	7.8
Mercury	13.6
Ethyl alcohol	0.81
Water	1.00
Sea water	1.03

and its units are N/m^2 in the SI system or Pascal. 1 Pa ≡ 1 N/m^2.

The pressure in any point in fluids depends on the depth of this point. Let us consider a fluid in a container, and let us choose a small element of this fluid with height $y_2 - y_1$ and area A as shown in Figure 13.1. Since this element of fluid is in equilibrium, the net forces acting on it must be zero. These forces are:

-upward force on the bottom of the sample $F_2=p_2A$

-downward force on the top is, $F_1=p_1A$

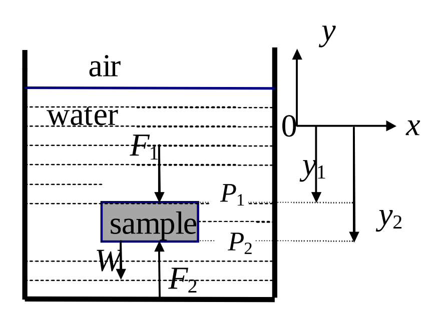


Figure 13.1 The forces acting on small element of the fluid. The sample is in equilibrium under these forces.

-the weight of the fluid element, $W = rg$ (volume of the sample) = rgA (y_1 -y_2). The resultant force in y-direction is zero, that is

$$\sum F_y = p_2 A - p_1 A - rgA(y_1 - y_2) = 0$$

or

$$p_2 = p_1 + r\,g(y_1 - y_2) \qquad (13.3)$$

To find the pressure at any point at a depth h in the fluid, (Figure 13.2) we choose level 1 on the surface where p_1 is the atmospheric pressure p_0, that is $p_2=p$, $p_1=p_0$, $y_1=0$, $y_2=-h$. Equation.13.3 becomes

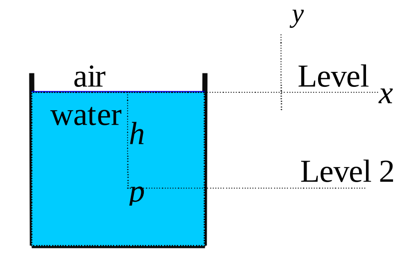


Figure 13.2 The pressure at any point at a depth h in the fluid.

$$p = p_0 + rgh \qquad (13.4)$$

Note that the pressure at any point inside the fluid consists of two parts: (1) p_o, the atmospheric pressure, (2) the pressure due to the liquid above this point. Also the pressure is the same at all points having the same elevation.

Example 13.1 U-tube contains two liquids in static equilibrium: Water and oil as shown in Fig.(13.3). Find the density of the oil. (ρ_{water}=1000 kg/m^3)

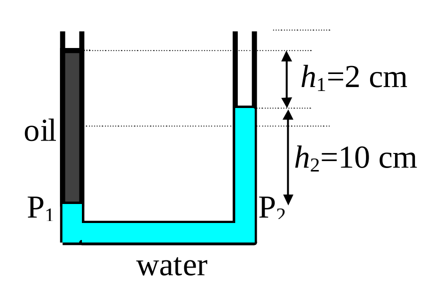


Figure 13.3 Example 13.1.

Solution The pressure at any level is the same in both arms of the U-tube. That is, the pressures at the interface sides are the same, i.e,

$$p_1 = p_2$$

$$r_{oil}\, g(h_1 + h_2) = r_{water}\, g h_2$$
$$r_{oil}\,(9.8)(0.12) = (1000)(9.8)(0.1)$$

$$r_{oil} = \frac{(1000)(9.8)(10\times!0^{-2})}{(9.8)(12\times10^{-2})}$$
$$= \frac{10000}{12} = 833.33\text{kg/m}^3$$

13.2 PASCAL'S PRINCIPLE

Pascal's principle states that, *a change in the pressur applied to an enclosed fluid is transmitted to every point of the fluid and the walls of the containing vessel.*

As an application of Pascal's principle, let us study the hydraulic lever shown in Figure 13.4. A small force F_1 is applied to a small piston of cross-sectional area A_1. The pressure is transmitted through the fluid to a large piston of cross-sectional area A_2. Since the pressure is the same on both sides, we have

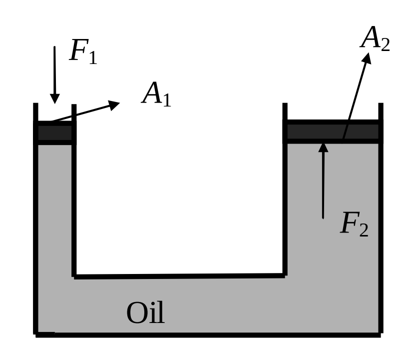
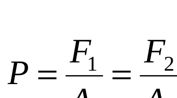


Figure 10.4 A hydroulic level, used to magnify a force F_1. Since the increase in pressure is the same at the left and right sides, a small force F_1 at the left produces a much larger force F_2 at the right.



$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

or

$$F_2 = F_1 \frac{A_2}{A_1}$$

Since $A_2 > A_1$, the resulting force F_2 is much larger than F_1. This principle is widely used, in hydraulic brakes, car lifts, hydraulic jacks, fork lifts, ITC.

Example 13.2 Calculate the force F_1 needed to lift a car weighing 12000 N, as shown in Figure 13.5. A_1=5 cm, and A_2=60 cm.

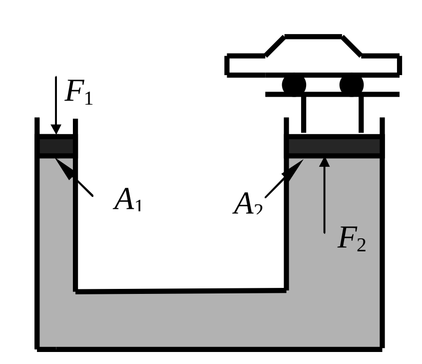


Figure 13.5 Example 13.2.

Solution Using the relation

$$F_1 = \frac{A_1}{A_2} F_2$$

we have

$$F_1 = \frac{5\times 10^{-2}}{60\times 10^{-2}} \times 12000 = 1000N$$

13.3 ARCHIMEDE'S PRINCIPLE

Archimede's principle states that; *any body completely or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body*.

Let us consider a cube of water totally submerged in a container filled with water as shown in Figure 13.6. This cube is in equilibrium under two forces. One of these forces is the weight, *mg*, of the cube of water acting downward, the second is the **buoyant**

force, *B*, acting upward. That is the buoyant force B equals in magnitude to the weight of the cube of water, or

$$B = mg \qquad (13.6)$$

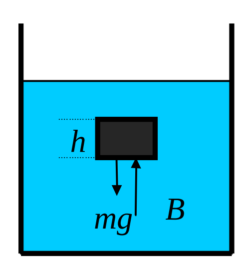


Figure 13.6 The external forces on the cube of water are its weight and the buoyancy force B. Under equilibrium conditions, the two forces are equal.

Now let us show that the buoyant force is equal to the weight of the displaced water. The pressure difference, *Dp*, between the top and the bottom of the cube is equal to the buoyant force *B* per unit area of the cube *A*, that is

$$\Delta p = \frac{B}{A},$$

or

$$B = \Delta p A = (r_w gh)A$$
$$= r_w gV = g(r_w V) = gM$$

where *M* is the mass of the cube which is equal to $r_w V$. Hence

B=Dp A= Mg = weight of the displaced water.

Example 13.3 A cube of wood 20 cm on a side and having a density of 0.65×10^3 kg/m^3 floats on water, as shown in Figure 13.7(a).

a) What is the distance from the top of the cube to the water level?

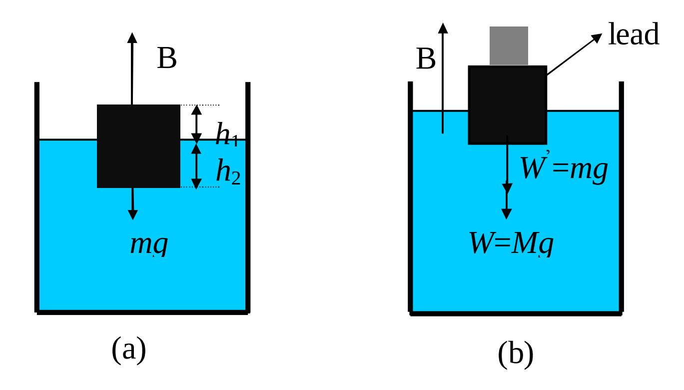


Figure 13.7 Example 13.3.

b) How much lead weight has to be placed on top of the cube so that its top is just level with the water?

Solution a) The cube of the wood is in equilibrium, so

$$r_w g(volume of\ displaced\ water)$$
$$= r_{wood}(volume of\ the cube)g$$

or

$$r_w g h_2 A = r_{wood}(h_1 + h_2)Ag$$

where A is the surface area of the cube. The last relation reduces to

$$r_w h_2 = r_{wood}(h_1 + h_2)$$
$$h_2 = \frac{(0.65\times10^3)(0.20)}{1000} = 0.130m,$$

and

$$h_1 = 0.20 - 0.130 = 0.07m.$$

b) In this case the volume of the displaced water is the same of the volume of the cube. If the weight of the lead is w', then

$$B = W^{\copyright} + Mg$$

or

$$W^{©} = B - Mg$$
$$= r_w g(volume of the displacede water) = (1000)(9.8)(0.20)^3$$
$$- r_{wood}(volume of the cube)g$$
$$-(0.65\times10^3)(0.20)^3(9.8) = (2.8)(9.8) \text{ N}$$

Hence, the lead mass, *m*=2.8 kg.

Example 13.4 A block of aluminum of mass 1 kg and density of 2.7×10^{3} kg/m^3 is suspended from a string and then completely immersed in water as shown in Figure 13.8. Calculate the tension in the string in the air and in the water.

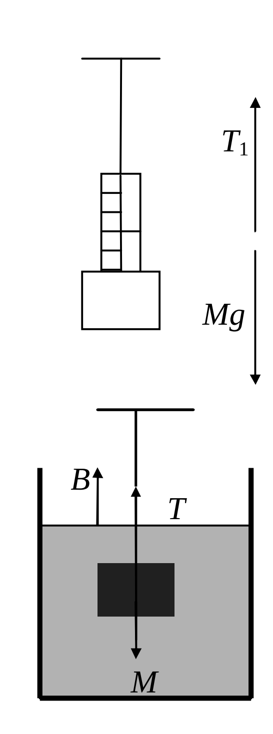


Figure 13.8 Example 13.4.

Solution In air the block is in equilibrium under two forces, the tension T_1 (the reading on the scale) and its weight *Mg*. (the buoyant force of the air can be neglect). That is,

$$T_1=Mg=1´9.8=9.8\ N.$$

In water, the block is balanced under three forces the tension T_2(the reading on the scale, the weight *Mg*, and the buoyant force upward of water. That is

$$T_2+B=Mg$$

$$T_2=Mg-B$$

The buoyant force B equals the weight of displaced water,

$$B = r_{water} V_{al} g = r_w \frac{M}{r_{al}} g$$

$$= 1 \times 10^3 \frac{1 \times 9.8}{2.7 \times 10^3} = 3.63N$$

Therefore

$$T_2 = Mg - B = 1 ´ 9.8 - 3.63 = 6.17 \text{ N}.$$

13.4 STREAMLINES AND THE CONTINUITY EQUATION

Streamlines

A streamline is the path taken by a fluid particle under steady flow. The velocity of any particle moves on a streamline is always tangent to these streamlines (see Figure 13.9). For a steady flow, no two streamlines can cross each other. Consider a non-uniform pipe with cross-sectional area A_1 at the bottom and A_2 at the top, as in Figure 13.10. In a time interval Δt, the fluid at the bottom end moves a distance $\Delta x_1 = v_1 \Delta t$. The mass of fluid in this part is $\Delta m_1 = r_1 A_1 \Delta x_1 = r_1 A_1 v_1 \Delta t$. Similarly, for the upper end of the pipe, we have $\Delta m_2 = r_2 A_2 v_2 \Delta t$. However, since the flow is steady, the

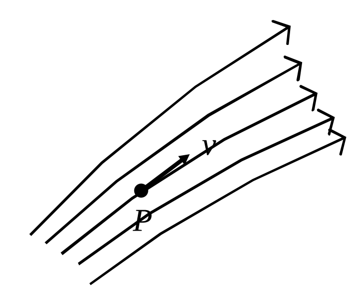


Figure 13.9 A set of steam lines. A particle P moves in one of these steam lines, and its velocity is tangent to the lines.

mass is conserved that is the mass crosses A_1 in a time interval Δt must equal the mass crosses A_2 in the same time interval Δt, i.e.,

$$Dm_1 = Dm_2$$

or

$$r_1 A_1 v_1 = r_2 A_2 v_2$$

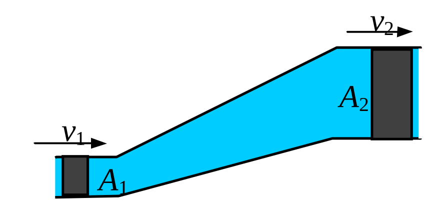


Figure 13.10 An incomprissible fluid with steady flow.

This is the equation of continuity. In incompressible fluid, the density ρ is always constant, then the above equation becomes

$$A_1 v_1 = A_2 v_2 \qquad (13.8)$$

That is the product of the area and the fluid speed at all points along the pipe is constant. The equation of continuity 13.8 can be expressed as

$$R = Av = \text{constant} \qquad (13.9)$$

where R is called the volume flow rate, A is the cross-sectional area of the tube at any point, and v is the speed of the fluid at that point.

13.5 BERNOUYLLI'S EQUATION

Consider a non-uniform tube through which a fluid is flowing at a steady rate. In a time interval Dt, suppose that a volume of fluid DV, (Figure 13.10(a)) enters the tube at the left (or input) end and an identical volume, emerges at the right (output) end (Figure 13.10(b)). The emerging volume must be the same as the entering

volume because the fluid is incompressible, with an assumed constant density ρ.

Let y_1, v_1, and p_1 be the elevation, speed, and pressure of the fluid entering at the left, and y_2, v_2, and p_2 be the corresponding quantities for the fluid emerging at the right. The force on the lower end of the fluid is p_1A_1, and the work done by this force is

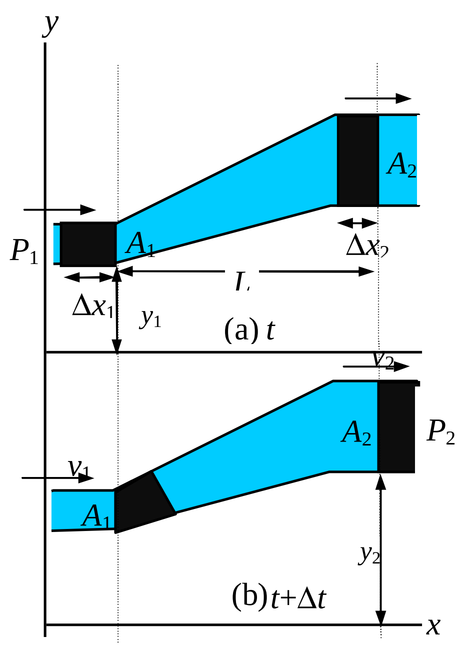


Figure 13.11

$$W_1=F_1\,Dx_1=p_1A_1Dx_1=p_1DV$$

In similar manner, the work done on the fluid at the upper end in the time Δt is given by

$$W_2=-p_2A_2Dx_2=-p_2DV$$

This work is negative since the fluid force opposes the displacement. Thus the net work done by these forces in the time interval Dt is

$$W=W_1+W_2=(p_1-p_2)\Delta V$$

Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy. The change in kinetic energy is given by

$$\Delta K=\frac{1}{2}(\Delta m)v_2^2-\frac{1}{2}(\Delta m)v_1^2$$

where *Dm* is the mass passing through the tube in the time interval *Dt.* And the change in potential energy is

$$\Delta U = \Delta mgy_2 - \Delta mgy_1$$

We can apply the work energy theorem in the form

$$W = \Delta K + \Delta V$$

to this volume of fluid to give

$$(p_1 - p_2)\Delta V$$
$$= \frac{1}{2}(\Delta m)v_2^2 - \frac{1}{2}(\Delta m)v_1^2 + \Delta mgy_2 - \Delta mgy_1$$

dividing by ΔV, and recall that the density $r = \frac{\Delta m}{\Delta V}$, the above expression reduces to

$$(p_1 - p_2) = \frac{1}{2} r v_2^2 - \frac{1}{2} r v_1^2 + r \quad gy_2 - r \quad gy_1$$

or

$$p_1 + \frac{1}{2} r v_1^2 + r \quad gy_1 = p_2 + \frac{1}{2} r v_2^2 + r \quad gy_2 \qquad (13.10)$$

This is Bernoulli's equation as applied to a non-viscous, incompressible fluid in steady flow. It is often expressed as

$$p + \frac{1}{2} r v^2 + r \quad gy = constant \qquad (13.11)$$

When the fluid is at rest, $v_1 = v_2 = 0$, equation (13.10) becomes

$$p_1 - p_2 = rg(y_2 - y_1) = rgh$$

Example 13.5 Ethanol of density ρ=791 kg/m^3 flows smoothly through a horizontal pipe that tapers in cross-sectional area of A_1=1.20×10^3 m^2 to A_2=A_1/2 as shown in Figure 13.12 . The pressure difference Δp between the ends of the pipe is 4120 Pa. What is the volume flow rate R of the ethanol ?

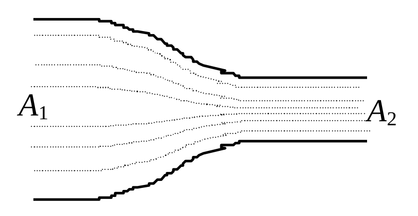


Figure 13.12 Example 13.5.

Solution From the equation of continuity, the volume flow rate (R = velocity × cross-sectional area) is the same in the wide and narrow sections. So

$$R = v_1 A_1 = v_2 A_2$$

where v_1 and v_2 are the velocities at the wide and narrow sections respectively.
Using A_2=A_1/2, we get

$$R = v_1 A_1 = v_2 \frac{A_1}{2},$$

$$v_1 = \frac{R}{A_1} \text{ and } v_2 = \frac{2R}{A_1}.$$

Bernoulli's equation can be written as

$$p_1 - p_2 = \frac{1}{2} r v_2^2 - \frac{1}{2} r v_1^2$$

Substituting v_1, v_2 and $\Delta p = p_1 - p_2$ into Bernoulli's equation, we get

$$\Delta p = \frac{1}{2} r\left(\frac{4R^2}{A_1^2} - \frac{R^2}{A_1^2}\right) = \frac{3rR^2}{2A_1^2}.$$

Solving for R, we find

$$R = A_1\sqrt{\frac{2\Delta p}{3r}}$$
$$= 1.20\times10^{-3}\sqrt{\frac{2\times4120}{3\times791}}$$
$$= 2.24\times10^{-3}\, m^3/s.$$

Example 13.6 A bullet is fired into an open water tank, Figure 13.13 , creating a hole a distance h below the water surface. What is the speed v of the water emerging from the hole?.

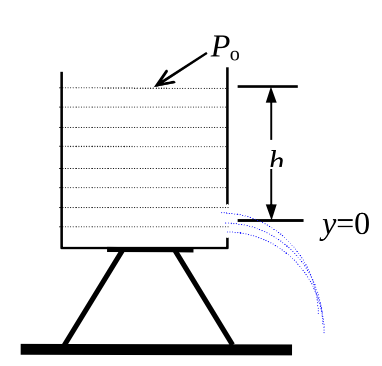


Figure 13.13 Example 13.6.

Solution This situation is essentially that of water moving (downward) with speed V through a wide pipe (the tank) of cross- sectional area A, and then moving (horizontally) with speed v through a narrow pipe (the hole) of cross-sectional area a. From the continuity equation, we know that

$$R = av = AV$$

and thus

$$V = \frac{a}{A} v$$

Because $a<<A$, we see that $V<<v$. We take the level of the hole as our reference level for measuring elevations (and thus gravitational potential energy.) Noting that the pressure at the top of the tank and at the bullet hole is the atmospheric pressure p_o (because both places are exposed to the atmosphere), we write Bernoulli's equation as

$$p_o + \frac{1}{2} rV^2 + rgh = p_o + \frac{1}{2} rv^2 + rg(0)$$

The zero on the right indicates that the hole is at our reference level. Using our result that $V<<v$, we assume that V^2 and thus the term $\frac{1}{2} rV^2$ is negligible compared to the other terms, and we drop it. Bernoulli's equation becomes

$$rgh = \frac{1}{2} rv^2$$

or

$$v = \sqrt{2gh} \, .$$

This is the same speed that an object would have when falling a height *h* from rest.

PROBLEMS

13.1 Find the mass of a solid iron sphere with diameter of 3.0 cm. The density of iron is $\rho = 7.86\times10^3$ kg/m^3

13.2 A gold crown has a mass of 0.5 kg and volume of 185 cm^3. Is the crown made of solid gold? The density of gold is $\rho = 19.3\times10^3$ kg/m^3.

13.3 At what depth in the sea is the absolute pressure equal three times atmospheric pressure?

13.4 The spring shown in Figure 13.14 has a force constant of 1000 N/m, and the piston has a diameter of 2 cm. Find the depth in water for which the spring compresses by 0.5 cm.

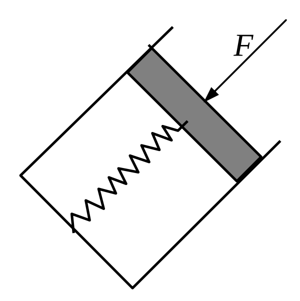


Figure 13.14 problem 13.4.

13.5 Find the pressure, in Pascal, 150 m below the surface of the ocean. The density of seawater is 1.03 g/cm^3 , and the atmospheric pressure at sea level is 1.01×10^5 Pa.

13.6 The human lungs can operate against a pressure differential of up to about 1/20 of an atmosphere. If a diver uses a snorkel for breathing, about how far below water level can he swim?

13.7 **a)** Find the total weight of water on top of a nuclear submarine at a depth of 200 m, assuming that its (horizontal cross-sectional) hull area is 3000 m^2.

b) What water pressure would a diver experience at this depth? Express your answer in atmospheres. Assume the density of seawater is 1.03 g/cm^3

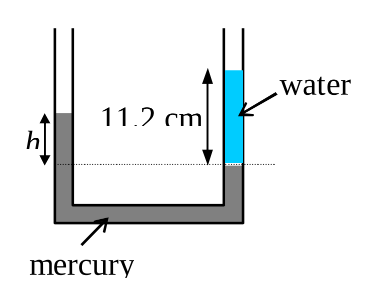


Figure 12.15 problem 12.8.

13.8 A sample, U-tube contains mercury. When 11.2 cm of water is poured into the right arm of the tube, how high above its initial level does the mercury rise in the left arm.

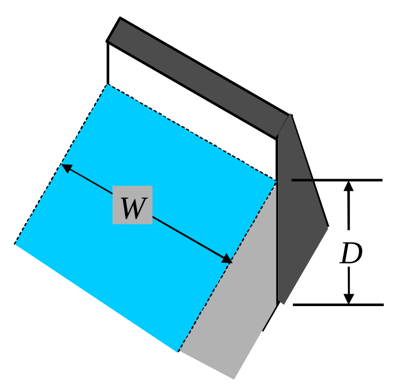
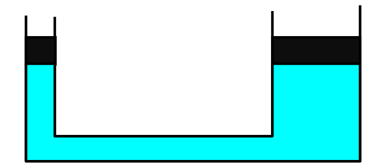


Figure 12.16problem 12.9.

13.9 Water stands at a depth *D* behind the vertical upstream face of a dam, as shown in Figure 13.16. The width of the dam is *W*. Find the horizontal force exerted on the dam by the gauge pressure of the water.


Figure 12.17 problem 12.10.

13.10 A piston of small cross-sectional area *a* is used in a hydraulic press to exert a small force *f* on the enclosed liquid. A connecting pipe leads to a larger piston of cross-sectional area *A* (Figure 13.17). If the small piston has a diameter of 3 cm, and the large piston has of 40 cm, what weight on the small piston will support 2.0 tons on the large piston.

13.11 A tin can has a total volume of 1200 cm^3 and a mass of 130 g. How many

grams of lead shot could it carry without sinking in water? The density of lead is 11.4 g/ cm^3 .

13.12 An ice cube floats in a glass of water as in Figure 13.17. What fraction of the ice cube lies above the water level?

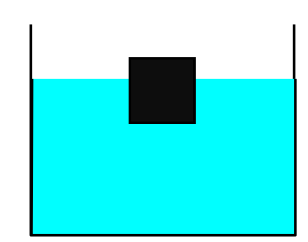


Figure 12.18 problem 12.12.

13.13 A cube of wood 20 cm on a side and having a density of 0.65×10^3 kg/m^3 floats on water.

a) What is the distance from the top of the cube to the water level?

b) How much lead weight has to be placed on top of the cube so that its top is just level with the water?

1.14 A sphere floats in water with 0.50 of its volume submerged. This same sphere floats in oil with 0.40 of its volume submerged. Determine the densities of the oil and the sphere.

13.15 An object hangs from a spring balance. The balance registers 30 N in air, 20 N when this object is immersed in water, and 24 N when the object is immersed in another liquid of unknown density. What is the density of that other liquid.

13.16 A hollow sphere of inner radius of 8.0 cm and outer radius of 9.0 cm floats half-submerged in a liqiud of density 800 kg/m^3.

a) What is the mass of the sphere?

b) Calculate the density of the material of which the sphere is made.

13.17 A block of wood has a mass of 3.67 kg and a density of 600 kg/ m^3 . It is to be loaded with lead so that it will float in water

with 0.90 of its volume immersed. What mass of lead is needed

a) if the lead is on top of the wood and

b) if the lead is attached below the wood? The density lead is 1.13×10^4 kg/m^3 .

13.18 A water hose 2 cm in diameter is used to fill a 20 liter bucket. If it takes 1 min to fill the bucket, what is the speed v at which the water leaves the hose?

13.19 The rate of flow of water through a horizontal pipe is 2 m^3 /min. Determine the velocity of flow at a pipe when the diameter of the pipe is (a) 10cm, (b) 5 cm.

13.20 If the speed of flow past the lower surface of an airplane wing is 110 m/s, what speed of flow over the upper surface will give a pressure difference of 900 Pa between upper and lower surfaces? Take the density of air to be 1.30×10^{-3} g/cm^3 .

13.21 A horizontal pipe 10 cm in diameter has a smooth reduction to a pipe 5 cm in diameter. If the pressure of the water in the larger pipe is 8×10^4 Pa and the pressure in the smaller pipe is6×10^4 Pa, at what rate does water flow throw the pipe?

13.22 Water flows through a horizontal pipe and is delivered into the atmosphere at a speed of 15 m/s as shown in Figure 13.18. The diameters of the left and right sections of the pipe are 5.0 cm and 3.0 cm, respectively. What volume of water is delivered into the atmosphere during a 10min period? What is the flow speed of the water in the left section of the pipe?